Approximate Tree Matching with pq-Grams

Nikolaus Augsten[a], **Michael Böhlen, Johann Gamper**
DIS - Center for Database and Information Systems
Free University of Bozen-Bolzano, Italy
www.inf.unibz.it

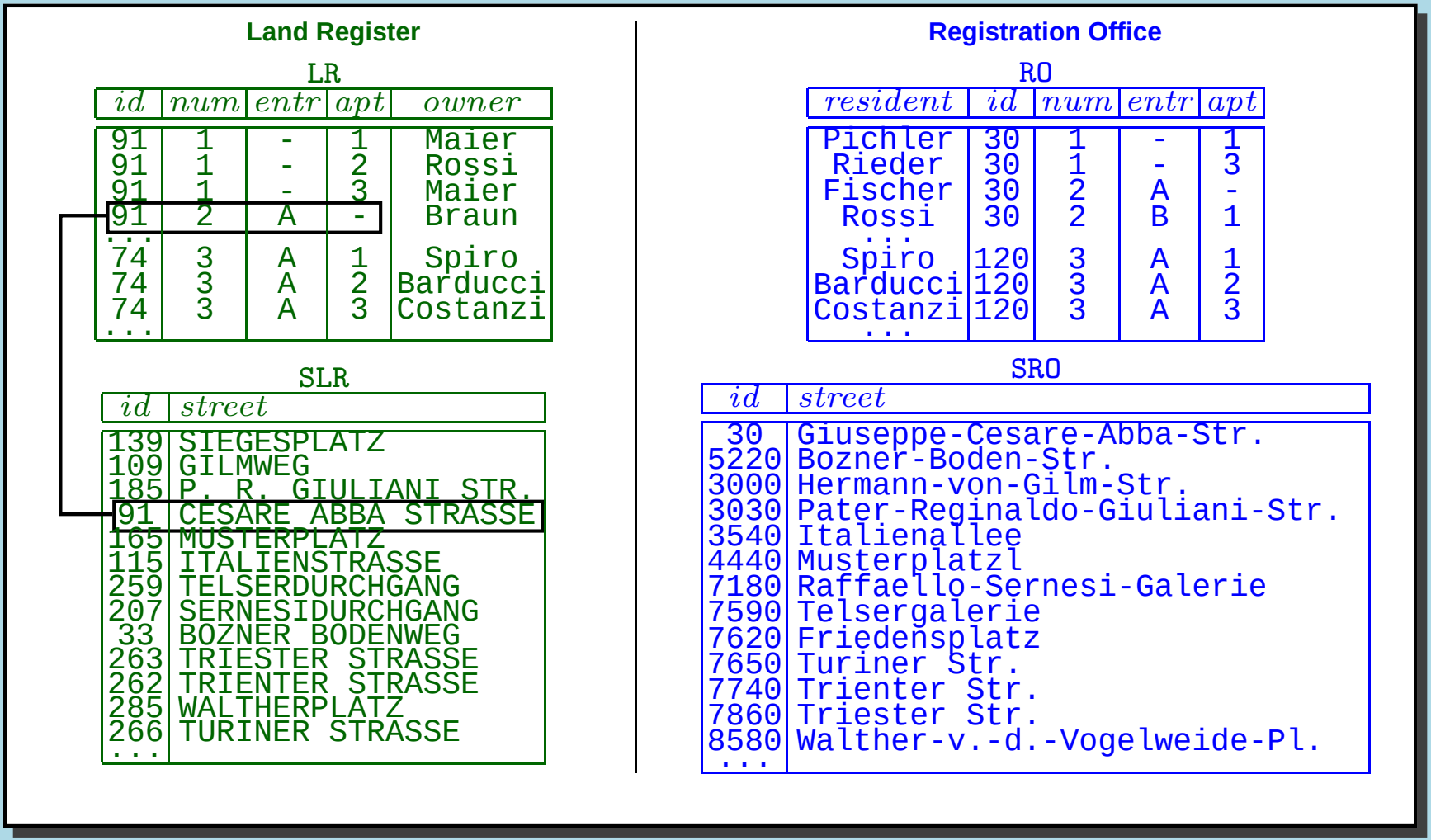
[a] Supported by the Municipality of Bozen-Bolzano.

Motivation — Example Data Sources

- We want to **link data items** in **different databases** that correspond to the **same real world object**.

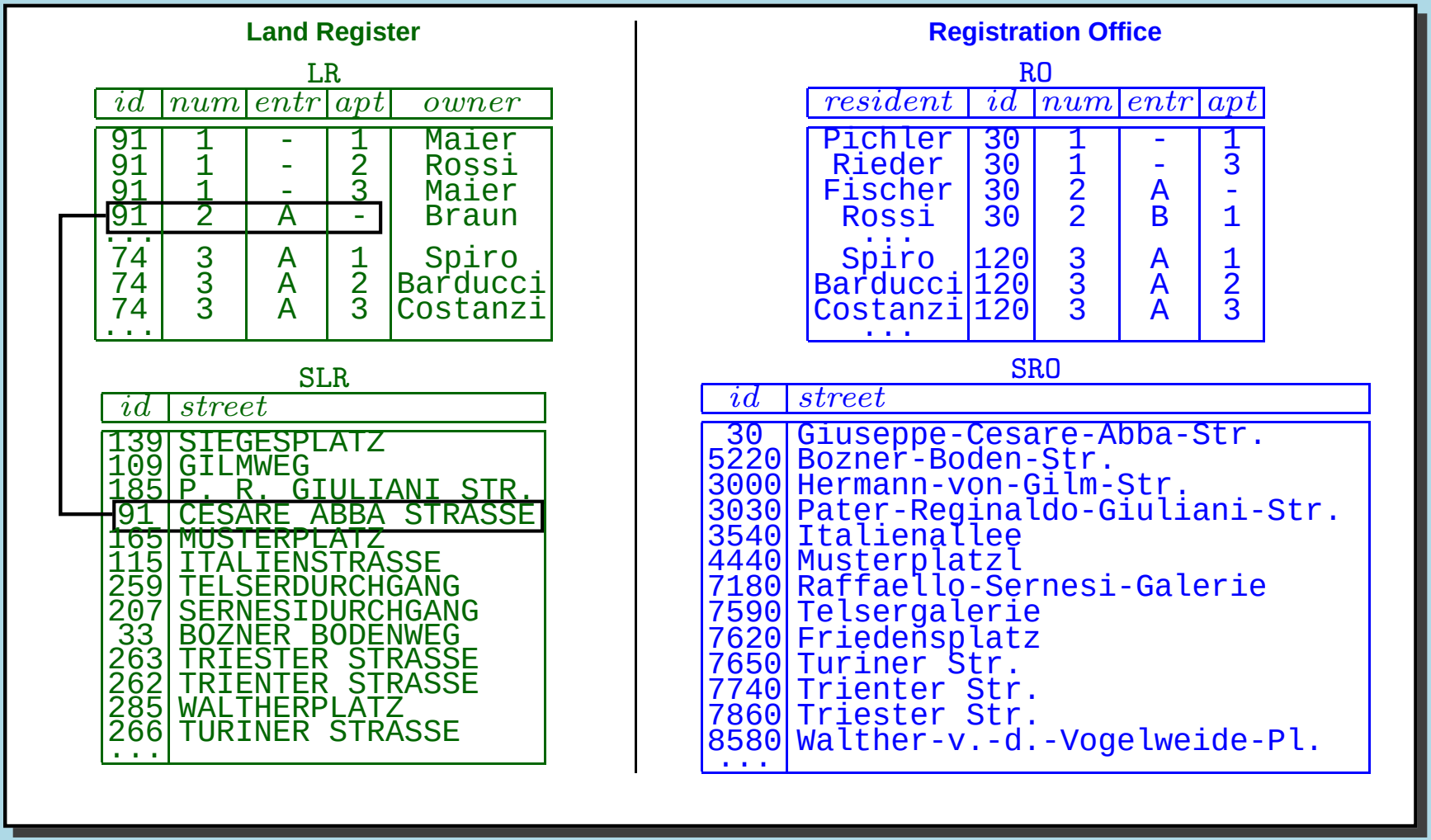
Motivation — Example Data Sources

☞ We want to **link data items** in **different databases** that correspond to the **same real world object**.



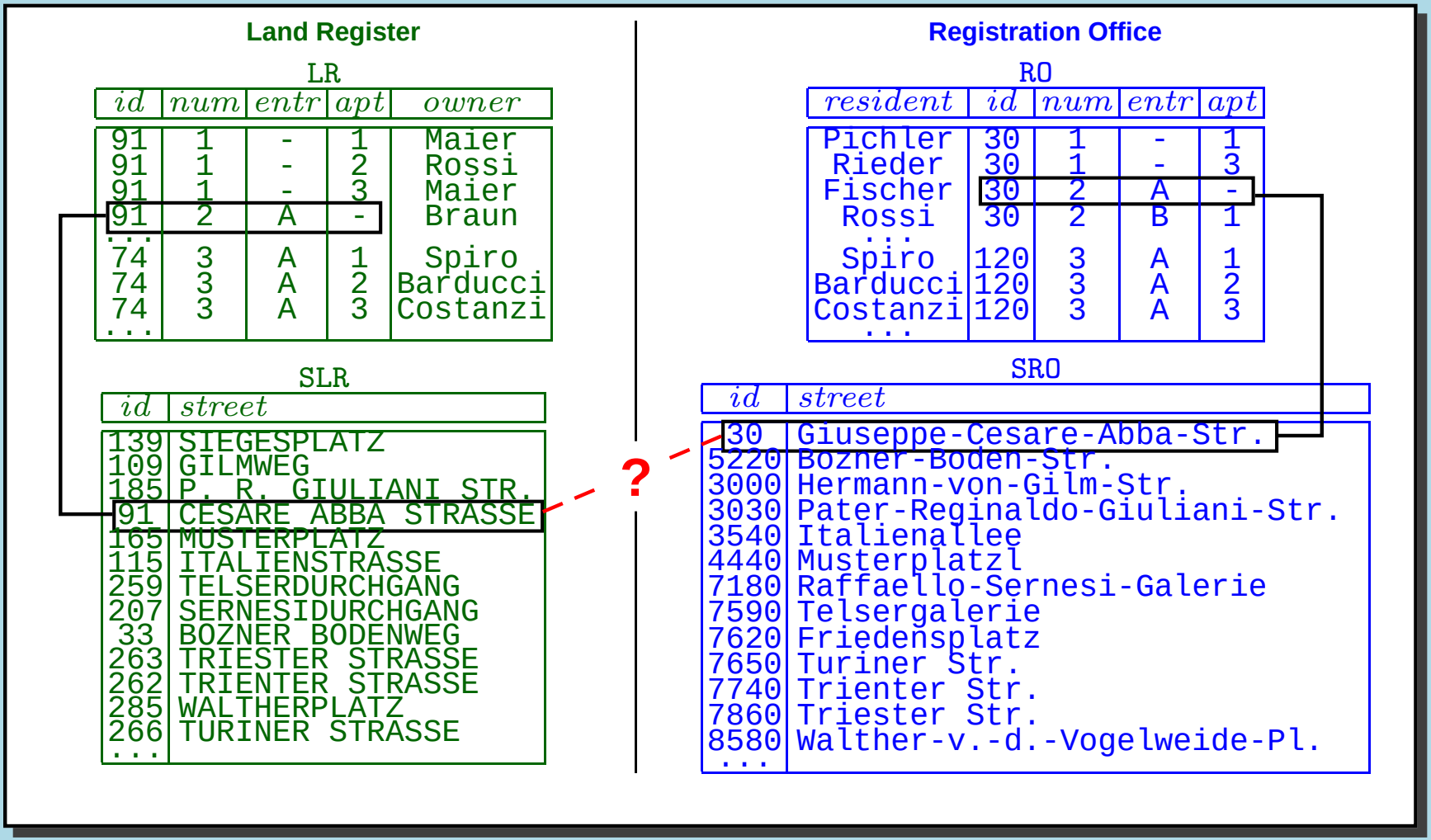
Motivation — Example Data Sources

- We want to **link data items** in **different databases** that correspond to the **same real world object**.
- **Example query:** Who lives in `Braun`'s apartment?



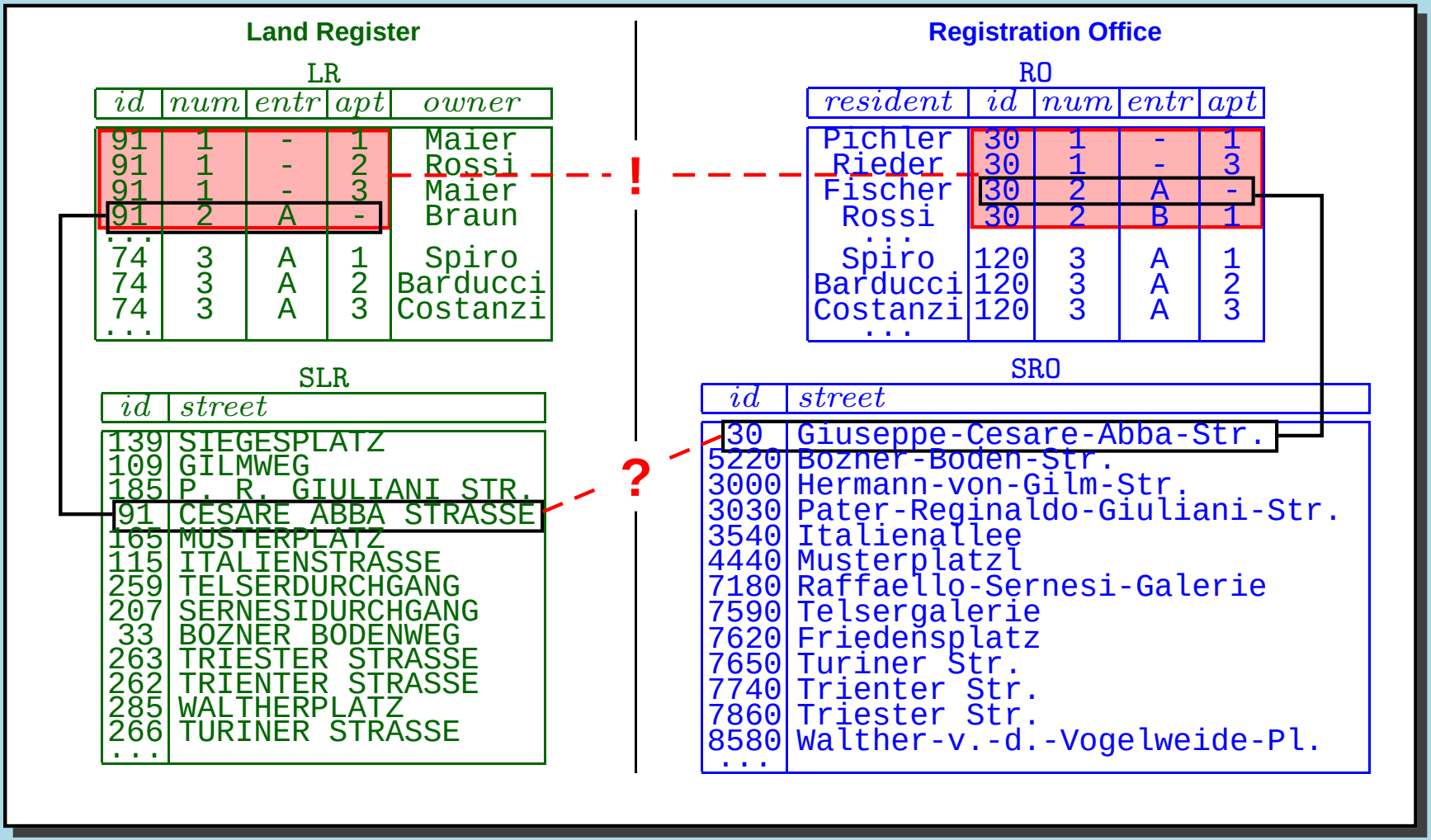
Motivation — Example Data Sources

- We want to **link data items** in **different databases** that correspond to the **same real world object**.
- **Example query:** Who lives in `Braun`'s apartment?



Motivation — Example Data Sources

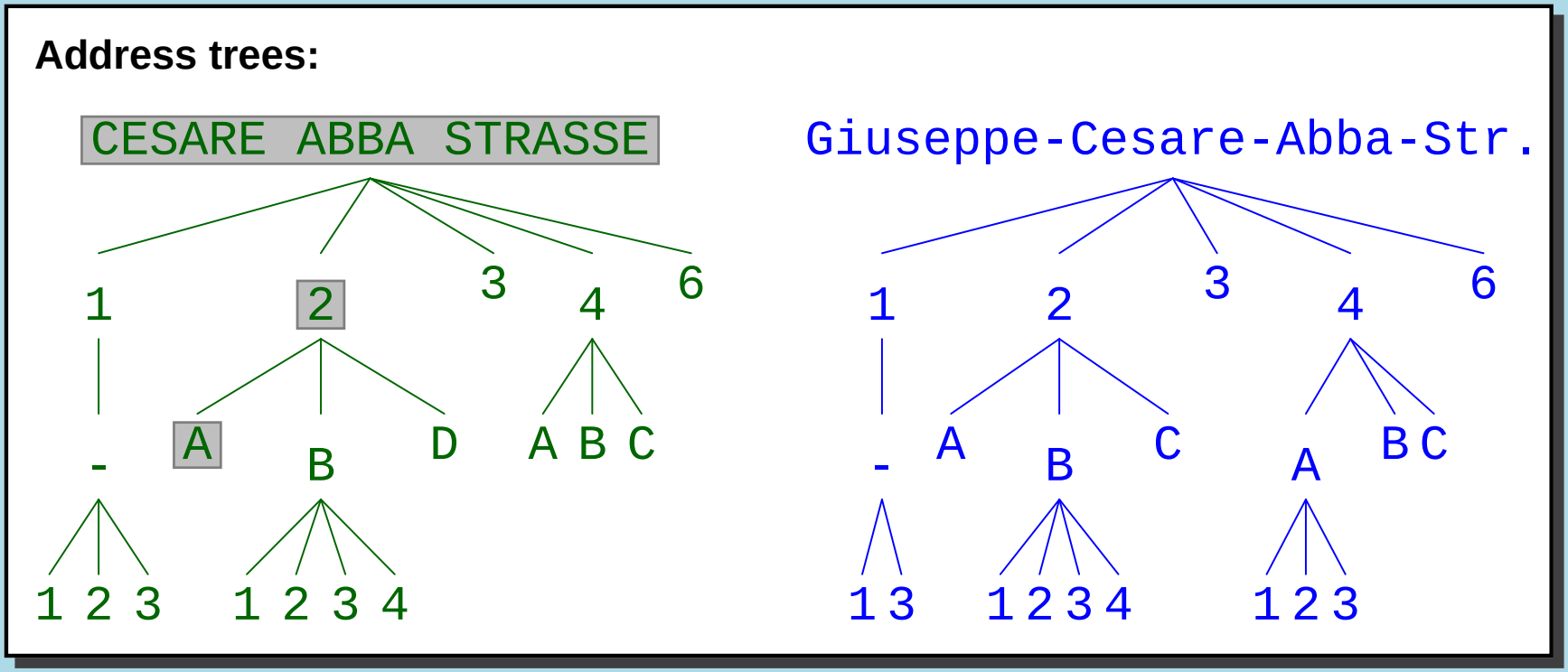
- We want to **link data items** in **different databases** that correspond to the **same real world object**.
- **Example query:** Who lives in `Braun`'s apartment?



Motivation — Address Trees

☞ residential addresses are hierarchical ⟶ **address tree**

Address trees:

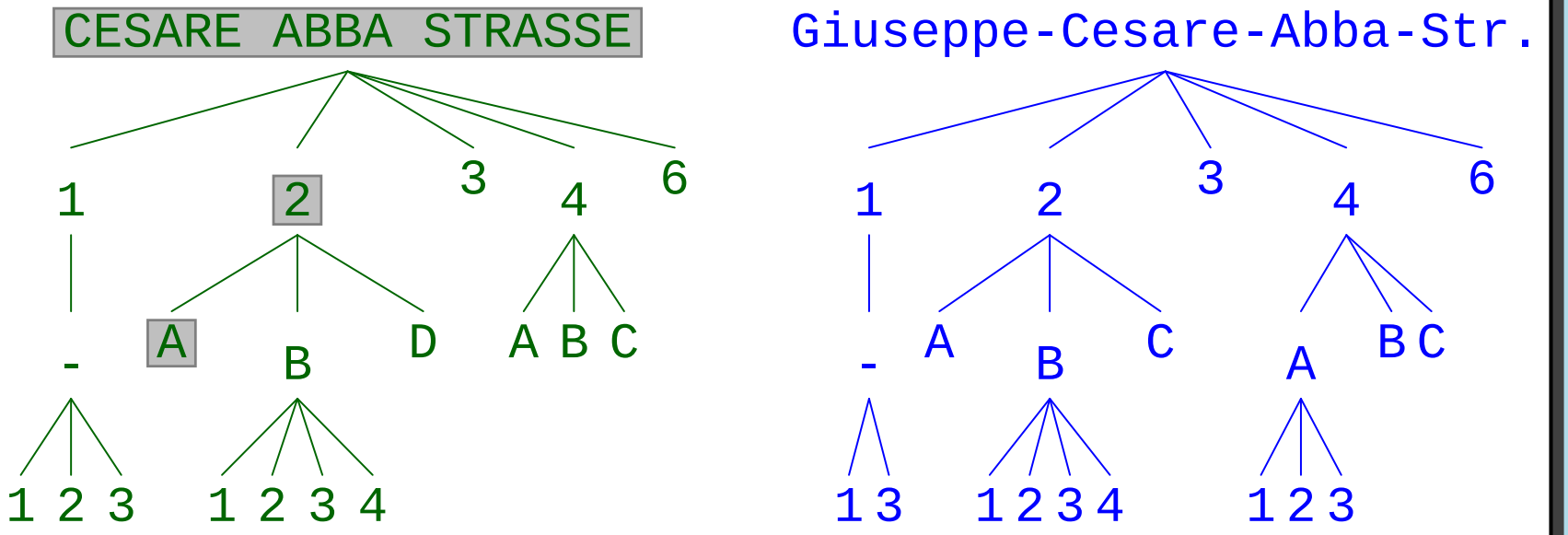


Motivation — Address Trees

- residential addresses are hierarchical ⟶ **address tree**
- **Idea:** corresponding streets ⇒ similar address tree

How similar are two address trees?

Address trees:

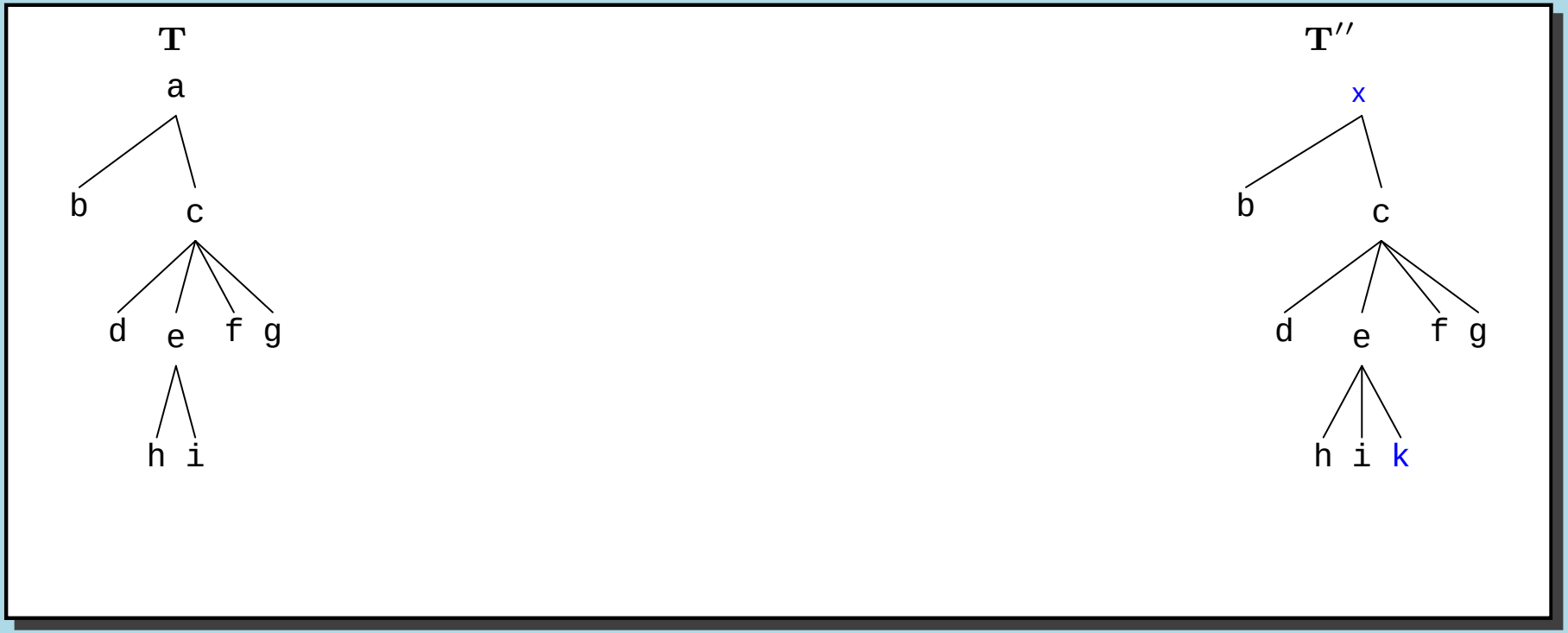


Motivation — Standard Solution: The Edit Distance

- **Edit distance:** Minimum cost sequence of edit operations (node insertion, node deletion, and label change) that transform one tree into an other.

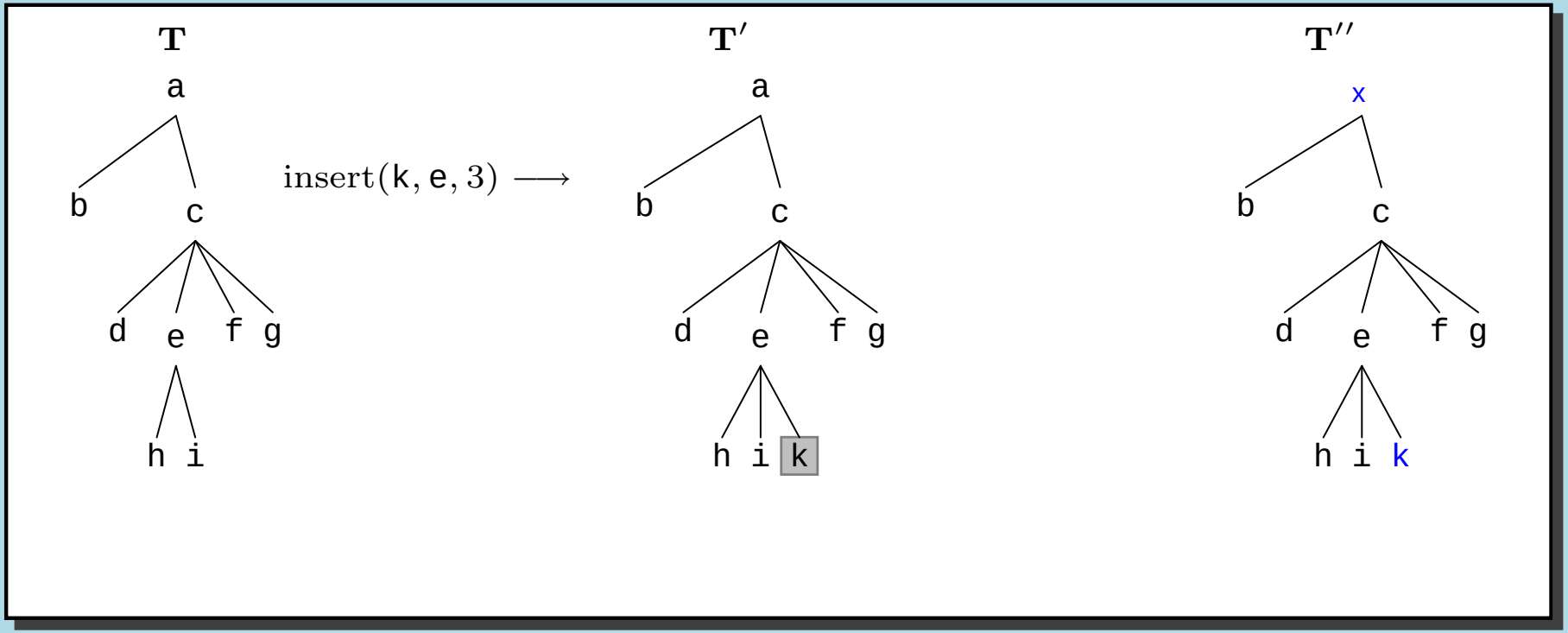
Motivation — Standard Solution: The Edit Distance

☞ **Edit distance:** Minimum cost sequence of edit operations (node insertion, node deletion, and label change) that transform one tree into an other.



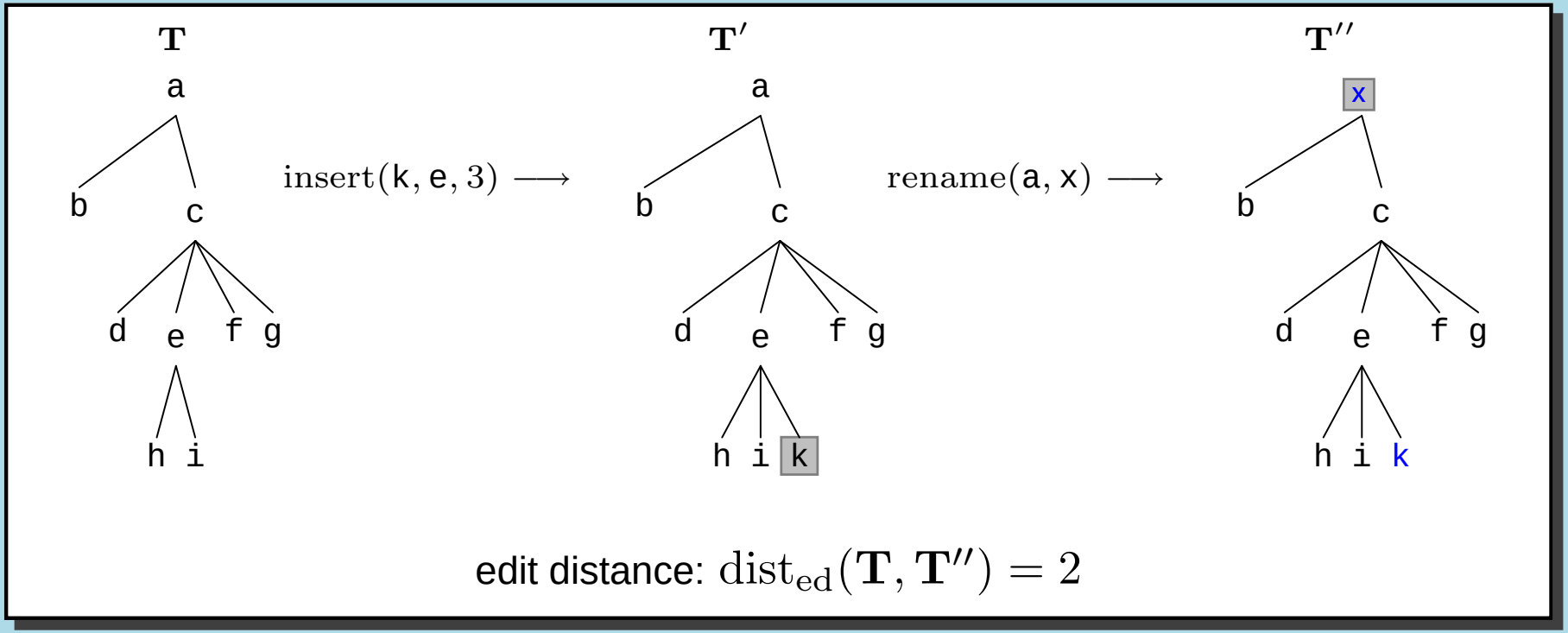
Motivation — Standard Solution: The Edit Distance

☞ **Edit distance:** Minimum cost sequence of edit operations (node insertion, node deletion, and label change) that transform one tree into an other.



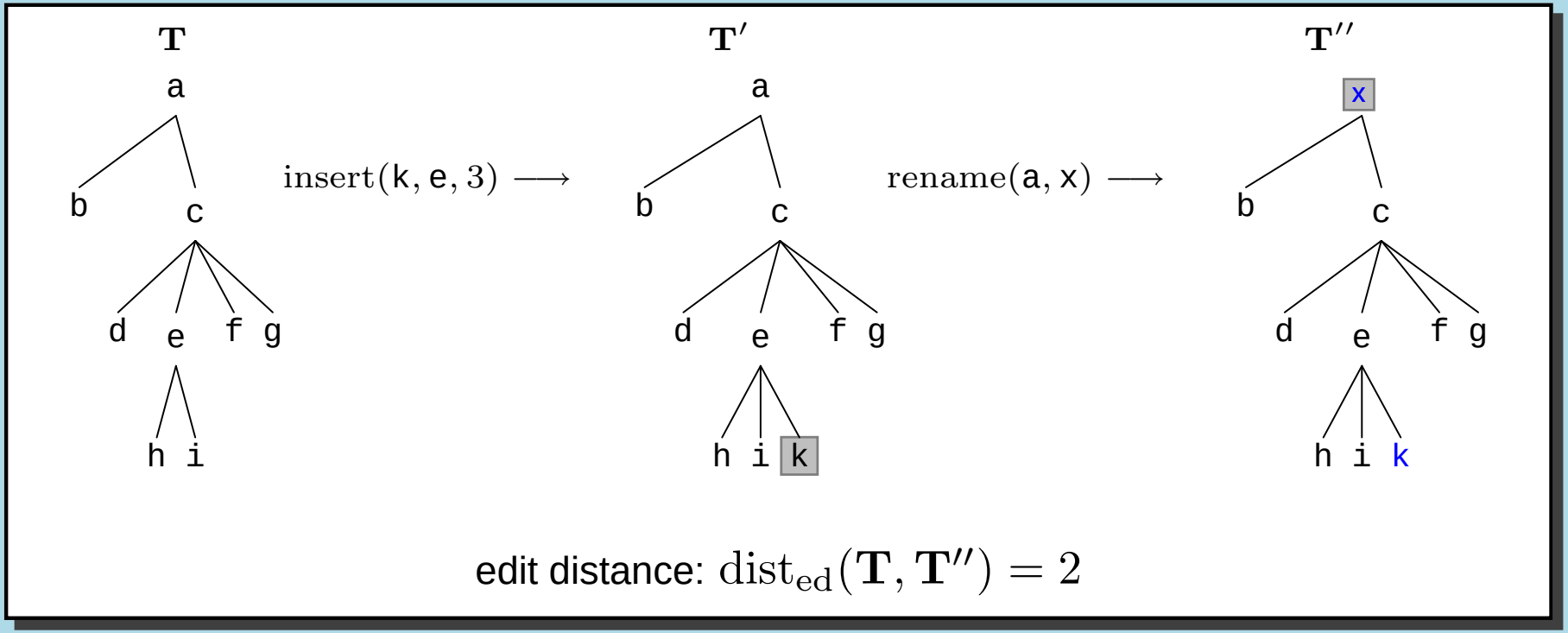
Motivation — Standard Solution: The Edit Distance

☞ **Edit distance:** Minimum cost sequence of edit operations (node insertion, node deletion, and label change) that transform one tree into an other.



Motivation — Standard Solution: The Edit Distance

☞ **Edit distance:** Minimum cost sequence of edit operations (node insertion, node deletion, and label change) that transform one tree into an other.



☞ **Problem:** Best algorithms $O(n^2 \log^2(n)) \Rightarrow$ not scalable.

Motivation — Problem Definition

- **Our goal:** Find an **efficient** and **effective approximation** of the tree edit distance that
  - is **scalable** for large trees,
  - emphasizes **structure**.

Related Work — Tree Distances

- $n \longrightarrow$ number of tree nodes
- **Tree edit distance:**
  - for balanced trees [Zhang and Shasha, 1989]: $O(n^2 \log^2(n))$
  - for arbitrary trees [Klein, 1998]: $O(n^3 \log(n))$

Related Work — Tree Distances

- $n \longrightarrow$ number of tree nodes
- **Tree edit distance:**
  - for balanced trees [Zhang and Shasha, 1989]: $O(n^2 \log^2(n))$
  - for arbitrary trees [Klein, 1998]: $O(n^3 \log(n))$
- **Tree edit distance approximations:**
  - Restricted versions of the tree edit distance:
    - Alignment [Jiang et al., 1995]: $O(n^2)$
    - Isolated subtree [Tanaka and Tanaka, 1988]: $O(n^2)$
    - Top-down [Selkow, 1977, Yang, 1991]: $O(n^2)$
    - Bottom-up [Valiente, 2001]: $O(n) \longrightarrow$ only very specific domains

Related Work — Tree Distances

- $n \longrightarrow$ number of tree nodes
- **Tree edit distance:**
  - for balanced trees [Zhang and Shasha, 1989]: $O(n^2 \log^2(n))$
  - for arbitrary trees [Klein, 1998]: $O(n^3 \log(n))$
- **Tree edit distance approximations:**
  - Restricted versions of the tree edit distance:
    - Alignment [Jiang et al., 1995]: $O(n^2)$
    - Isolated subtree [Tanaka and Tanaka, 1988]: $O(n^2)$
    - Top-down [Selkow, 1977, Yang, 1991]: $O(n^2)$
    - Bottom-up [Valiente, 2001]: $O(n) \longrightarrow$ only very specific domains
  - XML versioning [Chawathe et al., 1996, Chawathe and Garcia-Molina, 1997, Lee et al., 2004]: $O(n^2)$ for very different trees

Related Work — Tree Distances

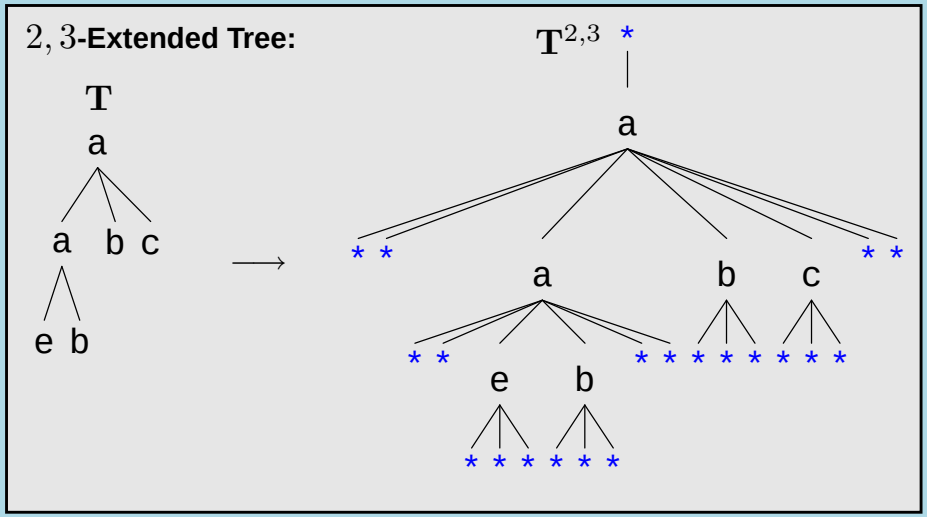
- $n \longrightarrow$ number of tree nodes
- **Tree edit distance:**
  - for balanced trees [Zhang and Shasha, 1989]: $O(n^2 \log^2(n))$
  - for arbitrary trees [Klein, 1998]: $O(n^3 \log(n))$
- **Tree edit distance approximations:**
  - Restricted versions of the tree edit distance:
    - Alignment [Jiang et al., 1995]: $O(n^2)$
    - Isolated subtree [Tanaka and Tanaka, 1988]: $O(n^2)$
    - Top-down [Selkow, 1977, Yang, 1991]: $O(n^2)$
    - Bottom-up [Valiente, 2001]: $O(n) \longrightarrow$ only very specific domains
  - XML versioning [Chawathe et al., 1996, Chawathe and Garcia-Molina, 1997, Lee et al., 2004]: $O(n^2)$ for very different trees
  - Tree-edit distance embedding [Garofalakis and Kumar, 2003, Garofalakis and Kumar, 2005]:
    - $O(n \log n)$
    - guaranteed distance distortion for tree edit distance with subtree move

Related Work — Tree Distances

- $n \longrightarrow$ number of tree nodes
- **Tree edit distance:**
  - for balanced trees [Zhang and Shasha, 1989]: $O(n^2 \log^2(n))$
  - for arbitrary trees [Klein, 1998]: $O(n^3 \log(n))$
- **Tree edit distance approximations:**
  - Restricted versions of the tree edit distance:
    - Alignment [Jiang et al., 1995]: $O(n^2)$
    - Isolated subtree [Tanaka and Tanaka, 1988]: $O(n^2)$
    - Top-down [Selkow, 1977, Yang, 1991]: $O(n^2)$
    - Bottom-up [Valiente, 2001]: $O(n) \longrightarrow$ only very specific domains
  - XML versioning [Chawathe et al., 1996, Chawathe and Garcia-Molina, 1997, Lee et al., 2004]: $O(n^2)$ for very different trees
  - Tree-edit distance embedding [Garofalakis and Kumar, 2003, Garofalakis and Kumar, 2005]:
    - $O(n \log n)$
    - guaranteed distance distortion for tree edit distance with subtree move
- **Related work for *strings*:**
  - Navarro [Navarro, 2001]: good overview of the edit distance for strings and its variants
  - Ukkonen [Ukkonen, 1992]: q-grams as lower bound for string edit distance

pq-Grams — Subtrees of the pq-Extended Tree

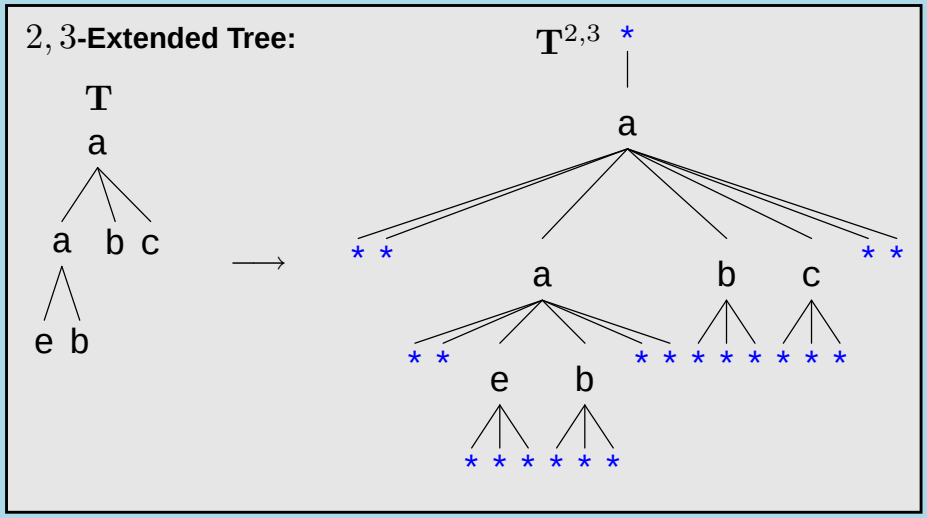
- **Extended Tree $\mathbf{T}^{pq}$:**
  Patch boundaries by adding null nodes (*):
  - $p-1$ ancestors to the root
  - $q-1$ nodes before the first and after the last child of each non-leaf node
  - q children to each leaf

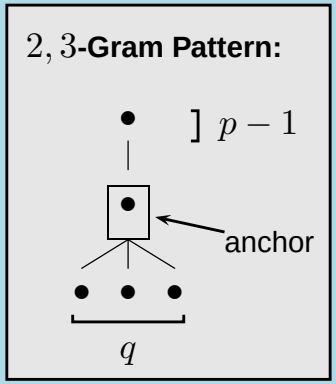


pq-Grams — Subtrees of the pq-Extended Tree

- **Extended Tree $\mathbf{T}^{pq}$:**
  Patch boundaries by adding null nodes (*):
  - $p-1$ ancestors to the root
  - $q-1$ nodes before the first and after the last child of each non-leaf node
  - q children to each leaf
- **pq-Gram $\mathbf{G}$:** Subtree of $\mathbf{T}^{pq}$.
  - Anchor node
  - with $p-1$ ancestors
  - and q children.

  Contiguous siblings in $\mathbf{G}$ are contiguous siblings in $\mathbf{T}^{pq}$.

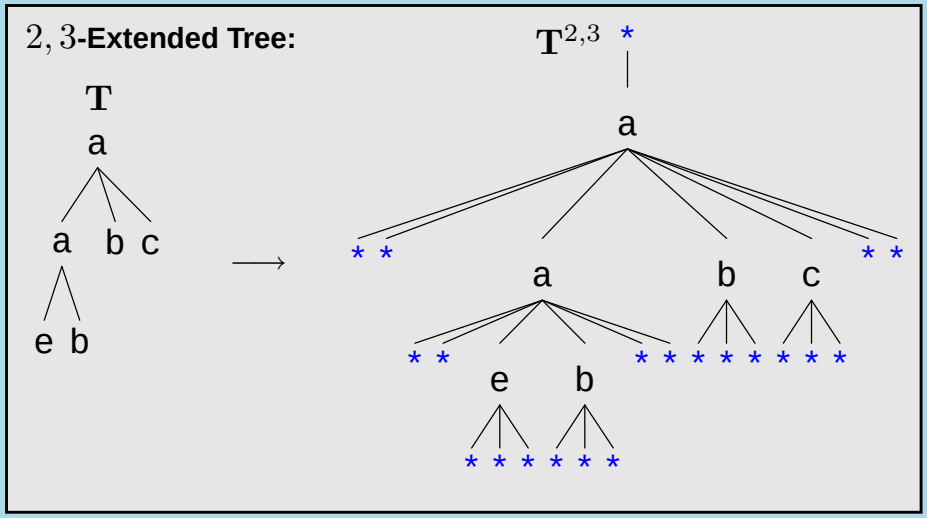


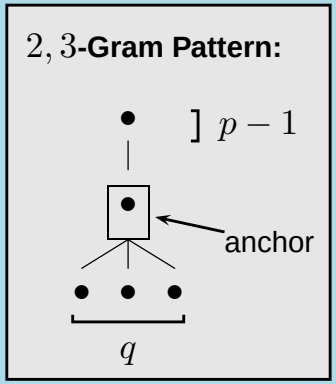


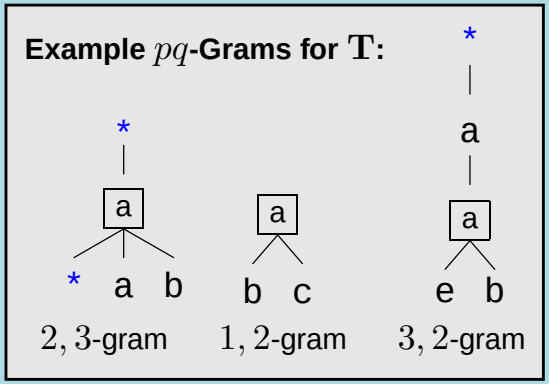
pq-Grams — Subtrees of the pq-Extended Tree

- **Extended Tree $\mathbf{T}^{pq}$:**
  Patch boundaries by adding null nodes (*):
  - $p-1$ ancestors to the root
  - $q-1$ nodes before the first and after the last child of each non-leaf node
  - q children to each leaf
- **pq-Gram $\mathbf{G}$:** Subtree of $\mathbf{T}^{pq}$.
  - Anchor node
  - with $p-1$ ancestors
  - and q children.

  Contiguous siblings in $\mathbf{G}$ are contiguous siblings in $\mathbf{T}^{pq}$.



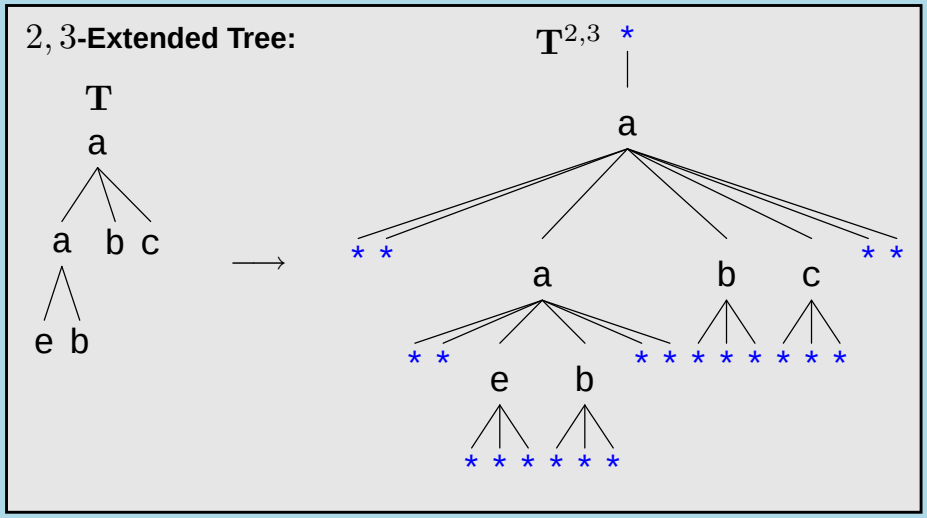


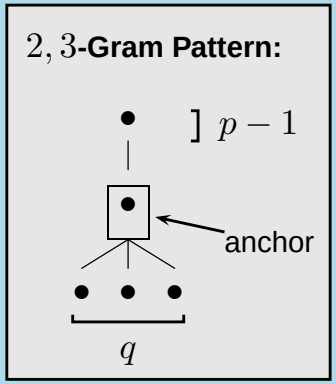


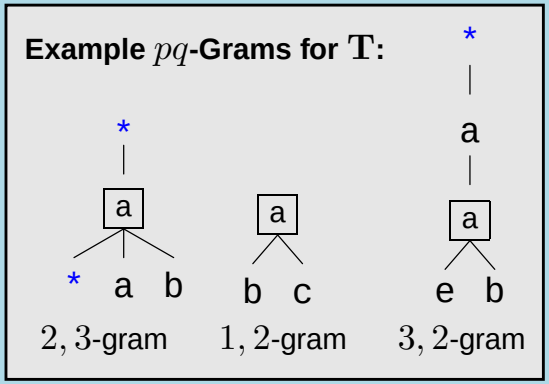
pq-Grams — Subtrees of the pq-Extended Tree

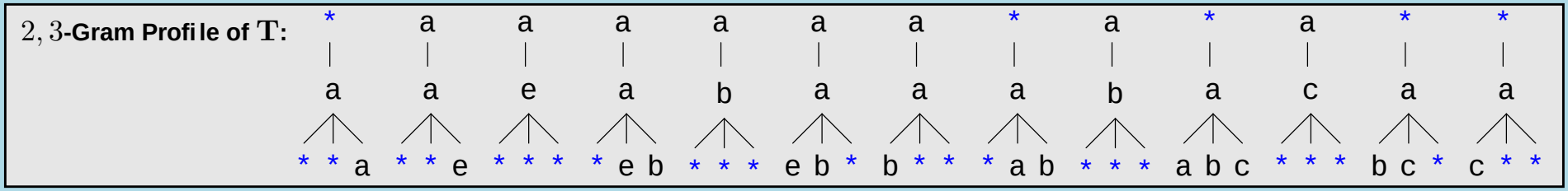
- **Extended Tree $\mathbf{T}^{pq}$:**
  Patch boundaries by adding null nodes (*):
  - $p-1$ ancestors to the root
  - $q-1$ nodes before the first and after the last child of each non-leaf node
  - q children to each leaf
- **pq-Gram $\mathbf{G}$:** Subtree of $\mathbf{T}^{pq}$.
  - Anchor node
  - with $p-1$ ancestors
  - and q children.

  Contiguous siblings in $\mathbf{G}$ are contiguous siblings in $\mathbf{T}^{pq}$.
- **pq-gram Profile $\mathrm{P}^{p,q}(\mathbf{T})$:**
  - Bag of all pq-grams of $\mathbf{T}$.



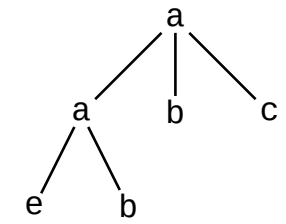






pq-Grams — Algorithm for pq-Gram Profile

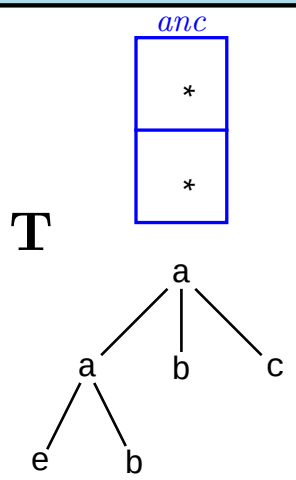
$\mathbf{T}$



P

```
1 createProfile(T, p, q, P, r, anc)
2    anc := shift(anc, l(r))
3    sib: shift register of size q (initialized with *)
4
5    if r is a leaf then
6      P := P ∪ (anc ∘ sib)
7    else
8      for each child c (from left to right) of r do
9        sib := shift(sib, l(c))
10       P := P ∪ (anc ∘ sib)
11       P := profile(T, p, q, P, c, anc)
12     for k := 1 to q − 1
13       sib := shift(sib, *)
14       P := P ∪ (anc ∘ sib)
15   return P
```

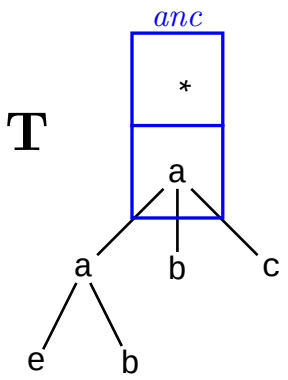
pq-Grams — Algorithm for pq-Gram Profile



P

```
1  CREATEPROFILE(T, p, q, P, r, anc)
2    anc := shift(anc, l(r))
3    sib: shift register of size q (initialized with *)
4
5    if r is a leaf then
6      P := P ∪ (anc ∘ sib)
7    else
8      for each child c (from left to right) of r do
9        sib := shift(sib, l(c))
10       P := P ∪ (anc ∘ sib)
11       P := PROFILE(T, p, q, P, c, anc)
12     for k := 1 to q − 1
13       sib := shift(sib, *)
14       P := P ∪ (anc ∘ sib)
15   return P
```

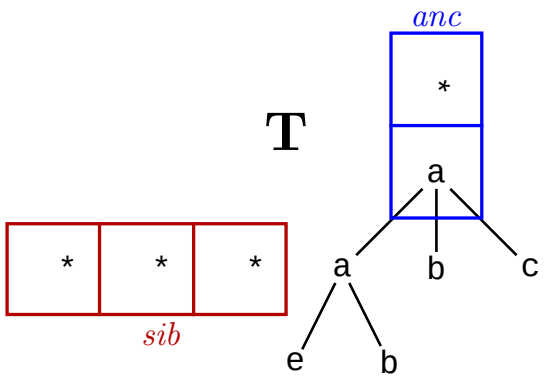
pq-Grams — Algorithm for pq-Gram Profile



P

```
1 CREATEPROFILE(T, p, q, P, r, anc)
2   anc := shift(anc, l(r))
3   sib: shift register of size q (initialized with *)
4
5   if r is a leaf then
6     P := P ∪ (anc ∘ sib)
7   else
8     for each child c (from left to right) of r do
9       sib := shift(sib, l(c))
10      P := P ∪ (anc ∘ sib)
11      P :=PROFILE(T, p, q, P, c, anc)
12    for k := 1 to q − 1
13      sib := shift(sib, *)
14      P := P ∪ (anc ∘ sib)
15  return P
```

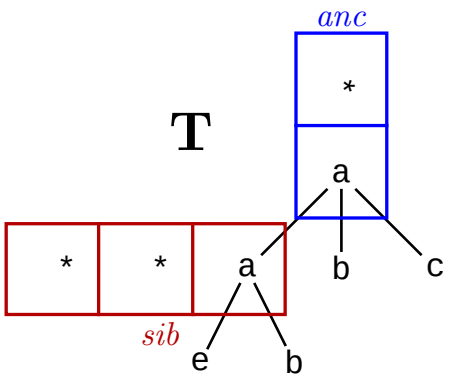
pq-Grams — Algorithm for pq-Gram Profile



P

```
1 CREATEPROFILE(T, p, q, P, r, anc)
2    anc := shift(anc, l(r))
3    sib: shift register of size q (initialized with *)
4
5    if r is a leaf then
6      P := P ∪ (anc ∘ sib)
7    else
8      for each child c (from left to right) of r do
9          sib := shift(sib, l(c))
10         P := P ∪ (anc ∘ sib)
11         P :=PROFILE(T, p, q, P, c, anc)
12     for k := 1 to q − 1
13         sib := shift(sib, *)
14         P := P ∪ (anc ∘ sib)
15   return P
```

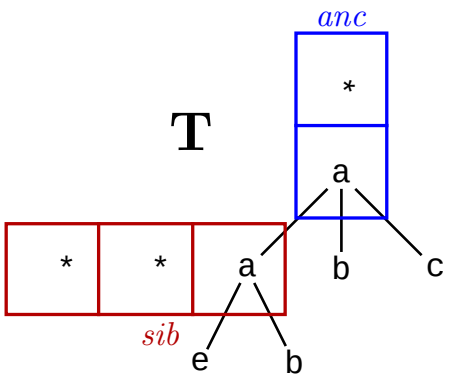
pq-Grams — Algorithm for pq-Gram Profile



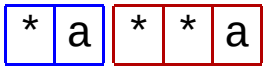
P

```
1 CREATEPROFILE(T, p, q, P, r, anc)
2    anc := shift(anc, l(r))
3    sib: shift register of size q (initialized with *)
4
5    if r is a leaf then
6      P := P ∪ (anc ∘ sib)
7    else
8      for each child c (from left to right) of r do
9        sib := shift(sib, l(c))
10       P := P ∪ (anc ∘ sib)
11       P :=PROFILE(T, p, q, P, c, anc)
12     for k := 1 to q − 1
13       sib := shift(sib, *)
14       P := P ∪ (anc ∘ sib)
15   return P
```

pq-Grams — Algorithm for pq-Gram Profile

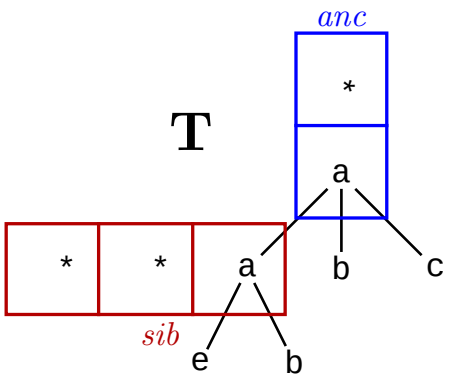


P

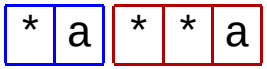



```
1  CREATEPROFILE(T, p, q, P, r, anc)
2     anc := shift(anc, l(r))
3     sib: shift register of size q (initialized with *)
4
5     if r is a leaf then
6       P := P ∪ (anc ∘ sib)
7     else
8       for each child c (from left to right) of r do
9           sib := shift(sib, l(c))
10          P := P ∪ (anc ∘ sib)
11          P := PROFILE(T, p, q, P, c, anc)
12    for k := 1 to q − 1
13          sib := shift(sib, *)
14          P := P ∪ (anc ∘ sib)
15    return P
```

pq-Grams — Algorithm for pq-Gram Profile

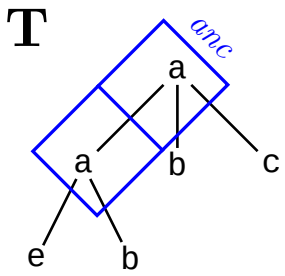


P

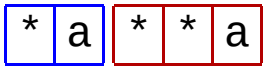



```
1  CREATEPROFILE(T, p, q, P, r, anc)
2     anc := shift(anc, l(r))
3     sib: shift register of size q (initialized with *)
4
5     if r is a leaf then
6       P := P ∪ (anc ∘ sib)
7     else
8       for each child c (from left to right) of r do
9           sib := shift(sib, l(c))
10          P := P ∪ (anc ∘ sib)
11          P := PROFILE(T, p, q, P, c, anc)
12    for k := 1 to q − 1
13          sib := shift(sib, *)
14          P := P ∪ (anc ∘ sib)
15    return P
```

pq-Grams — Algorithm for pq-Gram Profile

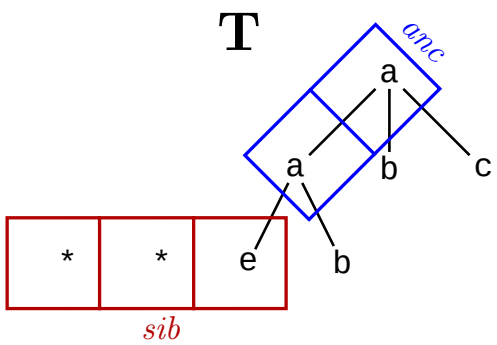


P

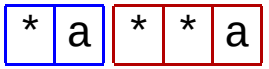



```
1 CREATEPROFILE(T, p, q, P, r, anc)
2   anc := shift(anc, l(r))
3   sib: shift register of size q (initialized with *)
4
5   if r is a leaf then
6     P := P ∪ (anc ∘ sib)
7   else
8     for each child c (from left to right) of r do
9       sib := shift(sib, l(c))
10      P := P ∪ (anc ∘ sib)
11      P := PROFILE(T, p, q, P, c, anc)
12    for k := 1 to q − 1
13      sib := shift(sib, *)
14      P := P ∪ (anc ∘ sib)
15  return P
```

pq-Grams — Algorithm for pq-Gram Profile

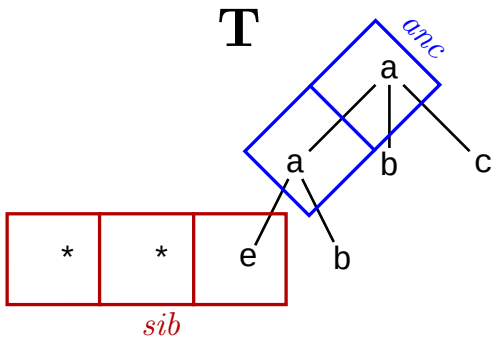


P

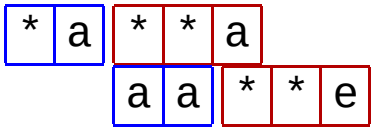



```
1  CREATEPROFILE(T, p, q, P, r, anc)
2     anc := shift(anc, l(r))
3     sib: shift register of size q (initialized with *)
4
5     if r is a leaf then
6       P := P ∪ (anc ∘ sib)
7     else
8       for each child c (from left to right) of r do
9           sib := shift(sib, l(c))
10          P := P ∪ (anc ∘ sib)
11          P := PROFILE(T, p, q, P, c, anc)
12    for k := 1 to q − 1
13          sib := shift(sib, *)
14          P := P ∪ (anc ∘ sib)
15    return P
```

pq-Grams — Algorithm for pq-Gram Profile

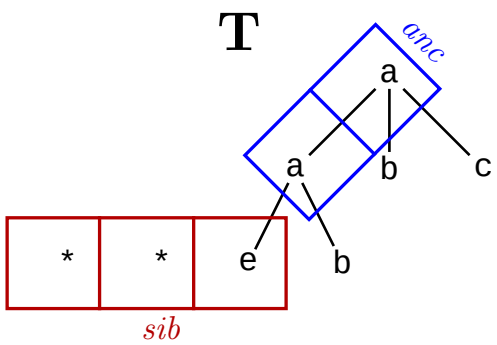


P

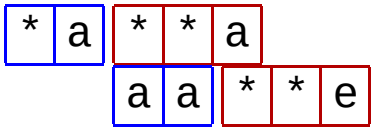



```
1  CREATEPROFILE(T, p, q, P, r, anc)
2     anc := shift(anc, l(r))
3     sib: shift register of size q (initialized with *)
4
5     if r is a leaf then
6       P := P ∪ (anc ∘ sib)
7     else
8       for each child c (from left to right) of r do
9           sib := shift(sib, l(c))
10          P := P ∪ (anc ∘ sib)
11          P := PROFILE(T, p, q, P, c, anc)
12    for k := 1 to q − 1
13          sib := shift(sib, *)
14          P := P ∪ (anc ∘ sib)
15    return P
```

pq-Grams — Algorithm for pq-Gram Profile

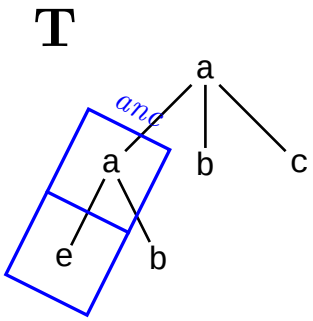


P

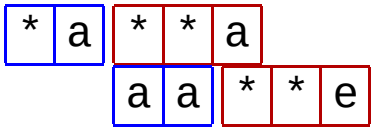



```
1  CREATEPROFILE(T, p, q, P, r, anc)
2     anc := shift(anc, l(r))
3     sib: shift register of size q (initialized with *)
4
5     if r is a leaf then
6       P := P ∪ (anc ∘ sib)
7     else
8       for each child c (from left to right) of r do
9           sib := shift(sib, l(c))
10          P := P ∪ (anc ∘ sib)
11          P := PROFILE(T, p, q, P, c, anc)
12    for k := 1 to q − 1
13          sib := shift(sib, *)
14          P := P ∪ (anc ∘ sib)
15    return P
```

pq-Grams — Algorithm for pq-Gram Profile

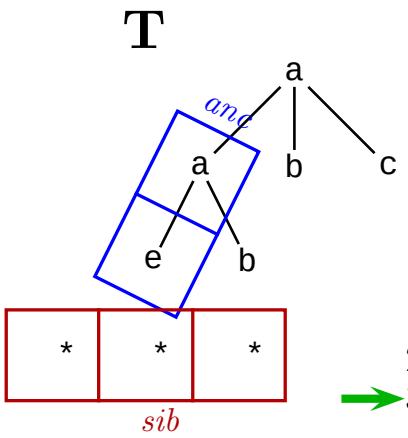


P



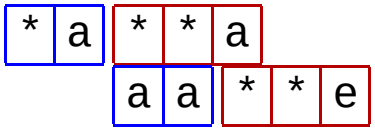

```
1 CREATEPROFILE(T, p, q, P, r, anc)
2   anc := shift(anc, l(r))
3   sib: shift register of size q (initialized with *)
4
5   if r is a leaf then
6     P := P ∪ (anc ∘ sib)
7   else
8     for each child c (from left to right) of r do
9       sib := shift(sib, l(c))
10      P := P ∪ (anc ∘ sib)
11      P := PROFILE(T, p, q, P, c, anc)
12    for k := 1 to q − 1
13      sib := shift(sib, *)
14      P := P ∪ (anc ∘ sib)
15  return P
```

pq-Grams — Algorithm for pq-Gram Profile

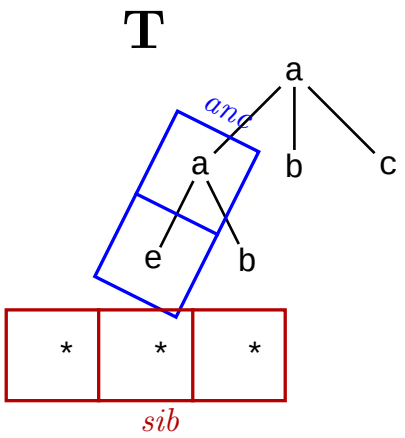



```
1 CREATEPROFILE(T, p, q, P, r, anc)
2    anc := shift(anc, l(r))
3    sib: shift register of size q (initialized with *)
4
5    if r is a leaf then
6      P := P ∪ (anc ∘ sib)
7    else
8      for each child c (from left to right) of r do
9          sib := shift(sib, l(c))
10         P := P ∪ (anc ∘ sib)
11         P :=PROFILE(T, p, q, P, c, anc)
12     for k := 1 to q − 1
13         sib := shift(sib, *)
14         P := P ∪ (anc ∘ sib)
15   return P
```

P

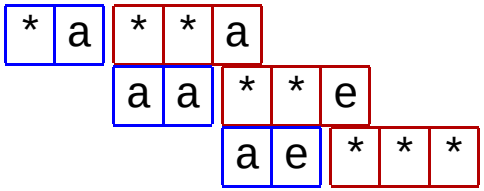


pq-Grams — Algorithm for pq-Gram Profile

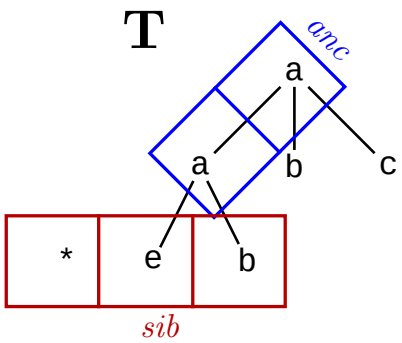



```
1 CREATEPROFILE(T, p, q, P, r, anc)
2    anc := shift(anc, l(r))
3    sib: shift register of size q (initialized with *)
4
5    if r is a leaf then
6       P := P ∪ (anc ∘ sib)
7    else
8       for each child c (from left to right) of r do
9          sib := shift(sib, l(c))
10         P := P ∪ (anc ∘ sib)
11         P :=PROFILE(T, p, q, P, c, anc)
12      for k := 1 to q − 1
13         sib := shift(sib, *)
14         P := P ∪ (anc ∘ sib)
15   return P
```

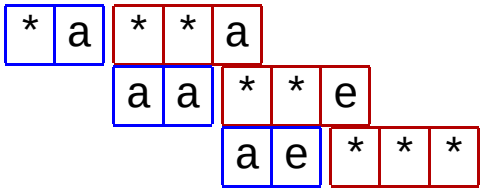
P



pq-Grams — Algorithm for pq-Gram Profile

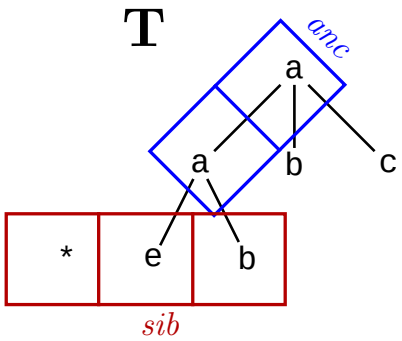


P




```
1  CREATEPROFILE(T, p, q, P, r, anc)
2     anc := shift(anc, l(r))
3     sib: shift register of size q (initialized with *)
4
5     if r is a leaf then
6       P := P ∪ (anc ∘ sib)
7     else
8       for each child c (from left to right) of r do
9           sib := shift(sib, l(c))
10          P := P ∪ (anc ∘ sib)
11          P := PROFILE(T, p, q, P, c, anc)
12    for k := 1 to q − 1
13          sib := shift(sib, *)
14          P := P ∪ (anc ∘ sib)
15    return P
```

pq-Grams — Algorithm for pq-Gram Profile

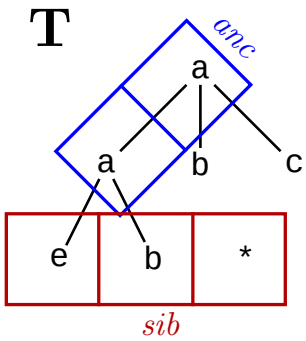



```
1  CREATEPROFILE(T, p, q, P, r, anc)
2     anc := shift(anc, l(r))
3     sib: shift register of size q (initialized with *)
4
5     if r is a leaf then
6       P := P ∪ (anc ∘ sib)
7     else
8       for each child c (from left to right) of r do
9           sib := shift(sib, l(c))
10          P := P ∪ (anc ∘ sib)
11          P := PROFILE(T, p, q, P, c, anc)
12    for k := 1 to q − 1
13          sib := shift(sib, *)
14          P := P ∪ (anc ∘ sib)
15    return P
```

P

*	a	*	*	a
a	a	*	*	e
		a	e	*	*	*
a	a	*	e	b
		a	b	*	*	*

pq-Grams — Algorithm for pq-Gram Profile

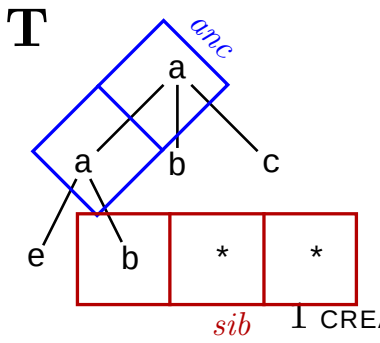



```
1  CREATEPROFILE(T, p, q, P, r, anc)
2     anc := shift(anc, l(r))
3     sib: shift register of size q (initialized with *)
4
5     if r is a leaf then
6       P := P ∪ (anc ∘ sib)
7     else
8       for each child c (from left to right) of r do
9           sib := shift(sib, l(c))
10          P := P ∪ (anc ∘ sib)
11          P := PROFILE(T, p, q, P, c, anc)
12      for k := 1 to q − 1
13          sib := shift(sib, *)
14          P := P ∪ (anc ∘ sib)
15    return P
```

P

*	a	*	*	a
a	a	*	*	e
		a	e	*	*	*
a	a	*	e	b
		a	b	*	*	*
a	a	e	b	*

pq-Grams — Algorithm for pq-Gram Profile

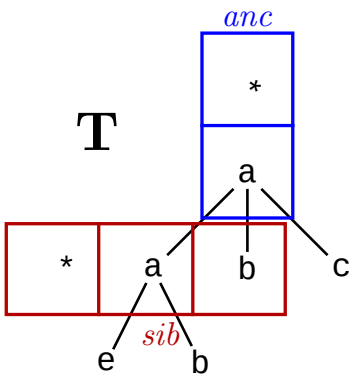



```
1  CREATEPROFILE(T, p, q, P, r, anc)
2    anc := shift(anc, l(r))
3    sib: shift register of size q (initialized with *)
4
5    if r is a leaf then
6      P := P ∪ (anc ∘ sib)
7    else
8      for each child c (from left to right) of r do
9        sib := shift(sib, l(c))
10       P := P ∪ (anc ∘ sib)
11       P :=PROFILE(T, p, q, P, c, anc)
12     for k := 1 to q − 1
13       sib := shift(sib, *)
14       P := P ∪ (anc ∘ sib)
15   return P
```

P

*	a	*	*	a
a	a	*	*	e
		a	e	*	*	*
a	a	*	e	b
		a	b	*	*	*
a	a	e	b	*
a	a	b	*	*

pq-Grams — Algorithm for pq-Gram Profile

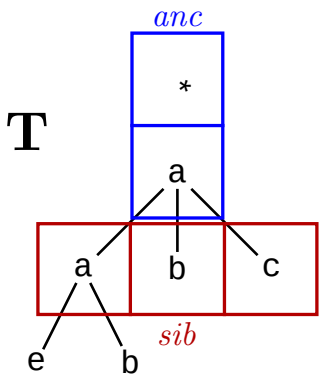



```
1  CREATEPROFILE(T, p, q, P, r, anc)
2     anc := shift(anc, l(r))
3     sib: shift register of size q (initialized with *)
4
5     if r is a leaf then
6       P := P ∪ (anc ∘ sib)
7     else
8       for each child c (from left to right) of r do
9           sib := shift(sib, l(c))
10          P := P ∪ (anc ∘ sib)
11          P := PROFILE(T, p, q, P, c, anc)
12    for k := 1 to q − 1
13          sib := shift(sib, *)
14          P := P ∪ (anc ∘ sib)
15    return P
```

P

*	a	*	*	a
a	a	*	*	e
a	e	*	*	*
a	a	*	e	b
a	b	*	*	*
a	a	e	b	*
a	a	b	*	*
*	a	*	a	b

pq-Grams — Algorithm for pq-Gram Profile

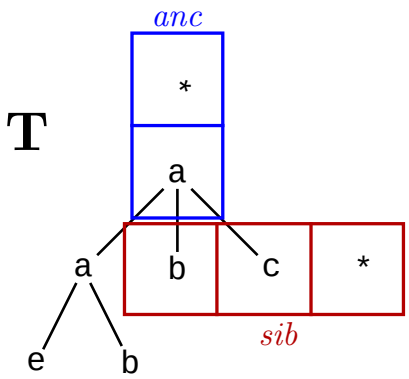



```
1  CREATEPROFILE(T, p, q, P, r, anc)
2     anc := shift(anc, l(r))
3     sib: shift register of size q (initialized with *)
4
5     if r is a leaf then
6       P := P ∪ (anc ∘ sib)
7     else
8       for each child c (from left to right) of r do
9           sib := shift(sib, l(c))
10          P := P ∪ (anc ∘ sib)
11          P := PROFILE(T, p, q, P, c, anc)
12    for k := 1 to q − 1
13          sib := shift(sib, *)
14          P := P ∪ (anc ∘ sib)
15    return P
```

P

*	a	*	*	a
a	a	*	*	e
a	e	*	*	*
a	a	*	e	b
a	b	*	*	*
a	a	e	b	*
a	a	b	*	*
*	a	*	a	b
a	b	*	*	*
*	a	a	b	c

pq-Grams — Algorithm for pq-Gram Profile

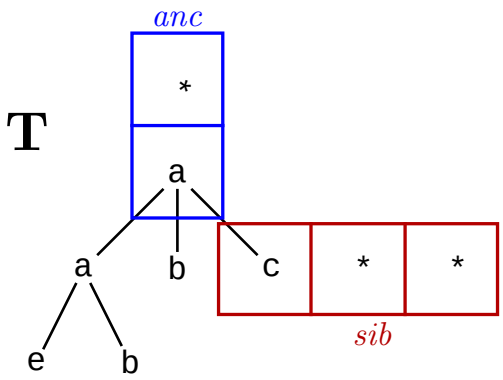



```
1  CREATEPROFILE(T, p, q, P, r, anc)
2     anc := shift(anc, l(r))
3     sib: shift register of size q (initialized with *)
4
5     if r is a leaf then
6        P := P ∪ (anc ∘ sib)
7     else
8        for each child c (from left to right) of r do
9           sib := shift(sib, l(c))
10          P := P ∪ (anc ∘ sib)
11          P := PROFILE(T, p, q, P, c, anc)
12       for k := 1 to q − 1
13          sib := shift(sib, *)
14          P := P ∪ (anc ∘ sib)
15    return P
```

P

*	a	*	*	a
a	a	*	*	e
a	e	*	*	*
a	a	*	e	b
a	b	*	*	*
a	a	e	b	*
a	a	b	*	*
*	a	*	a	b
a	b	*	*	*
*	a	a	b	c
a	c	*	*	*
*	a	b	c	*

pq-Grams — Algorithm for pq-Gram Profile

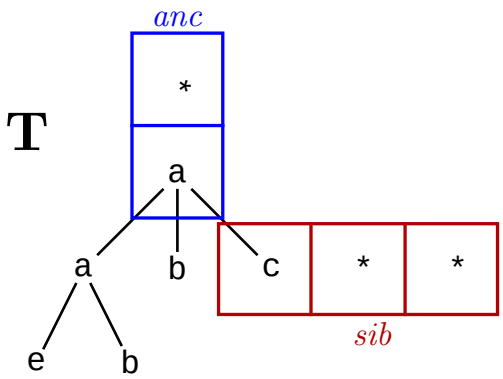



```
1  CREATEPROFILE(T, p, q, P, r, anc)
2     anc := shift(anc, l(r))
3     sib: shift register of size q (initialized with *)
4
5     if r is a leaf then
6       P := P ∪ (anc ∘ sib)
7     else
8       for each child c (from left to right) of r do
9           sib := shift(sib, l(c))
10          P := P ∪ (anc ∘ sib)
11          P := PROFILE(T, p, q, P, c, anc)
12    for k := 1 to q − 1
13          sib := shift(sib, *)
14          P := P ∪ (anc ∘ sib)
15    return P
```

P

*	a	*	*	a
		a	a	*	*	e
				a	e	*	*	*
		a	a	*	e	b
				a	b	*	*	*
		a	a	e	b	*
		a	a	b	*	*
*	a	*	a	b
		a	b	*	*	*
*	a	a	b	c
		a	c	*	*	*
*	a	b	c	*
*	a	c	*	*

pq-Grams — Algorithm for pq-Gram Profile




```
1  CREATEPROFILE(T, p, q, P, r, anc)
2     anc := shift(anc, l(r))
3     sib: shift register of size q (initialized with *)
4
5     if r is a leaf then
6       P := P ∪ (anc ∘ sib)
7     else
8       for each child c (from left to right) of r do
9           sib := shift(sib, l(c))
10          P := P ∪ (anc ∘ sib)
11          P := PROFILE(T, p, q, P, c, anc)
12      for k := 1 to q − 1
13          sib := shift(sib, *)
14          P := P ∪ (anc ∘ sib)
15    return P
```

P

*	a	*	*	a
		a	a	*	*	e
				a	e	*	*	*
		a	a	*	e	b
				a	b	*	*	*
		a	a	e	b	*
		a	a	b	*	*
*	a	*	a	b
		a	b	*	*	*
*	a	a	b	c
		a	c	*	*	*
*	a	b	c	*
*	a	c	*	*

pq-Grams — The pq-Gram Profile

The pq-gram profile is

☞ **small** $\longrightarrow$ size $O(n)$

Theorem 1 *For tree* $\mathbf{T}$ *with* l *leaves and* i *non-leaves:*

$$|\,\mathrm{P}^{p,q}(\mathbf{T})| = 2l + qi - 1.$$

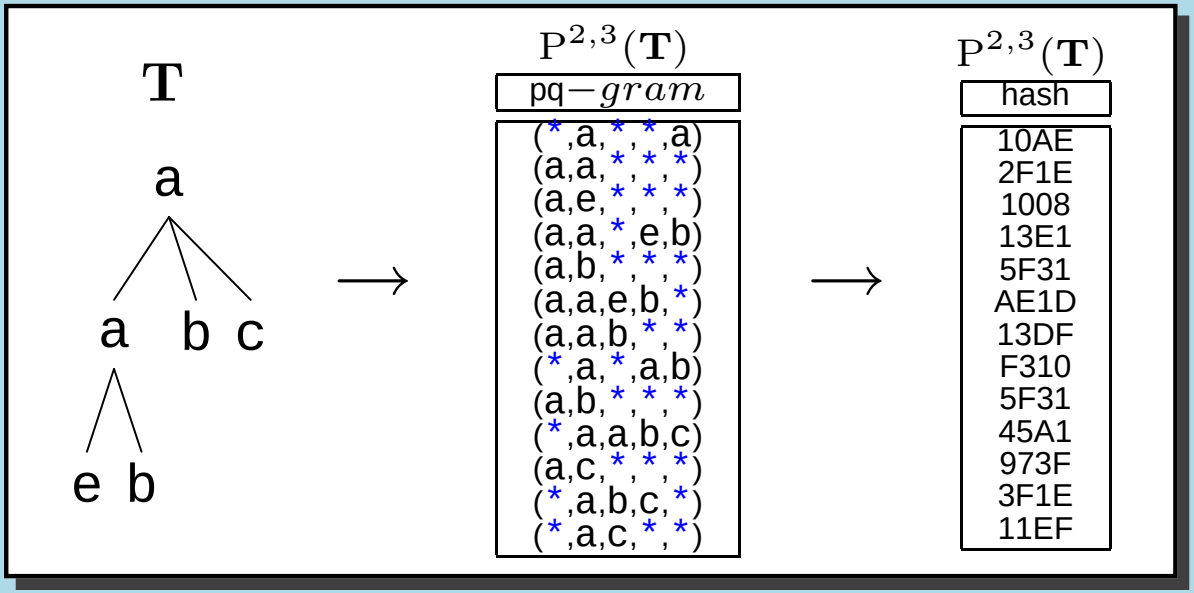
pq-Grams — The pq-Gram Profile

The pq-gram profile is

- **small** $\longrightarrow$ size $O(n)$
- **easy to store**
  - represent the pq-grams by fingerprint **hash value**
  - store profile in **single-attribute relation**

Theorem 1 *For tree* $\mathbf{T}$ *with* l *leaves and* i *non-leaves:*

$$|\mathrm{P}^{p,q}(\mathbf{T})| = 2l + qi - 1.$$



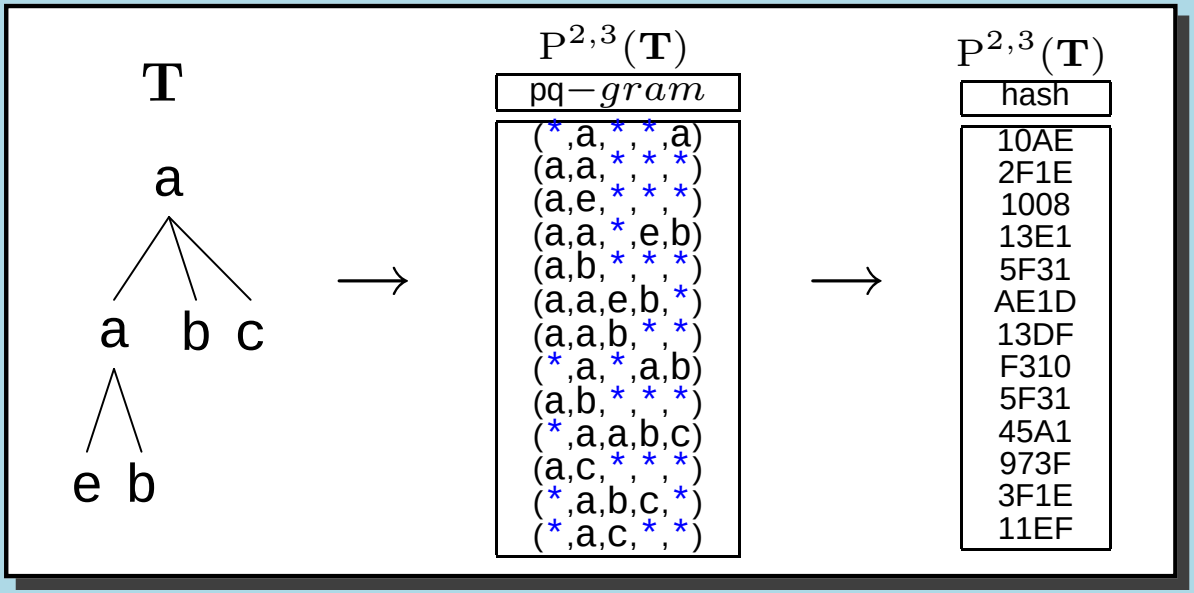
pq-Grams — The pq-Gram Profile

The pq-gram profile is

- **small** $\longrightarrow$ size $O(n)$
- **easy to store**
  - represent the pq-grams by fingerprint **hash value**
  - store profile in **single-attribute relation**
- **allows** effective **distance computation** between trees

Theorem 1 *For tree* $\mathbf{T}$ *with* l *leaves and* i *non-leaves:*

$$|\mathrm{P}^{p,q}(\mathbf{T})| = 2l + qi - 1.$$



☞ **Definition 1** *For two trees* $\mathbf{T}_1$ *and* $\mathbf{T}_2$ *the* ***pq*-gram distance** *is:*

$$\Delta^{p,q}(\mathbf{T}_1, \mathbf{T}_2) = 1 - 2\frac{|\,\mathrm{P}^{p,q}(\mathbf{T}_1) \cap \mathrm{P}^{p,q}(\mathbf{T}_2)|}{|\,\mathrm{P}^{p,q}(\mathbf{T}_1) \cup \mathrm{P}^{p,q}(\mathbf{T}_2)|}$$

☞ **Definition 1** *For two trees* $\mathbf{T}_1$ *and* $\mathbf{T}_2$ *the* ***pq*-gram distance** *is:*

$$\Delta^{p,q}(\mathbf{T}_1, \mathbf{T}_2) = 1 - 2\frac{|\,\mathrm{P}^{p,q}(\mathbf{T}_1) \cap \mathrm{P}^{p,q}(\mathbf{T}_2)|}{|\,\mathrm{P}^{p,q}(\mathbf{T}_1) \cup \mathrm{P}^{p,q}(\mathbf{T}_2)|}$$

☞ can be computed in $O(n \log n)$ time and $O(n)$ space (bag intersection of relations)

☞ **Definition 1** *For two trees* $\mathbf{T}_1$ *and* $\mathbf{T}_2$ *the* ***pq*-gram distance** *is:*

$$\Delta^{p,q}(\mathbf{T}_1, \mathbf{T}_2) = 1 - 2\frac{|\,\mathrm{P}^{p,q}(\mathbf{T}_1) \cap \mathrm{P}^{p,q}(\mathbf{T}_2)|}{|\,\mathrm{P}^{p,q}(\mathbf{T}_1) \cup \mathrm{P}^{p,q}(\mathbf{T}_2)|}$$

(bag intersection of relations)

☞ can be computed in $O(n \log n)$ time and $O(n)$ space

☞ other terms are constants for **normalization**:

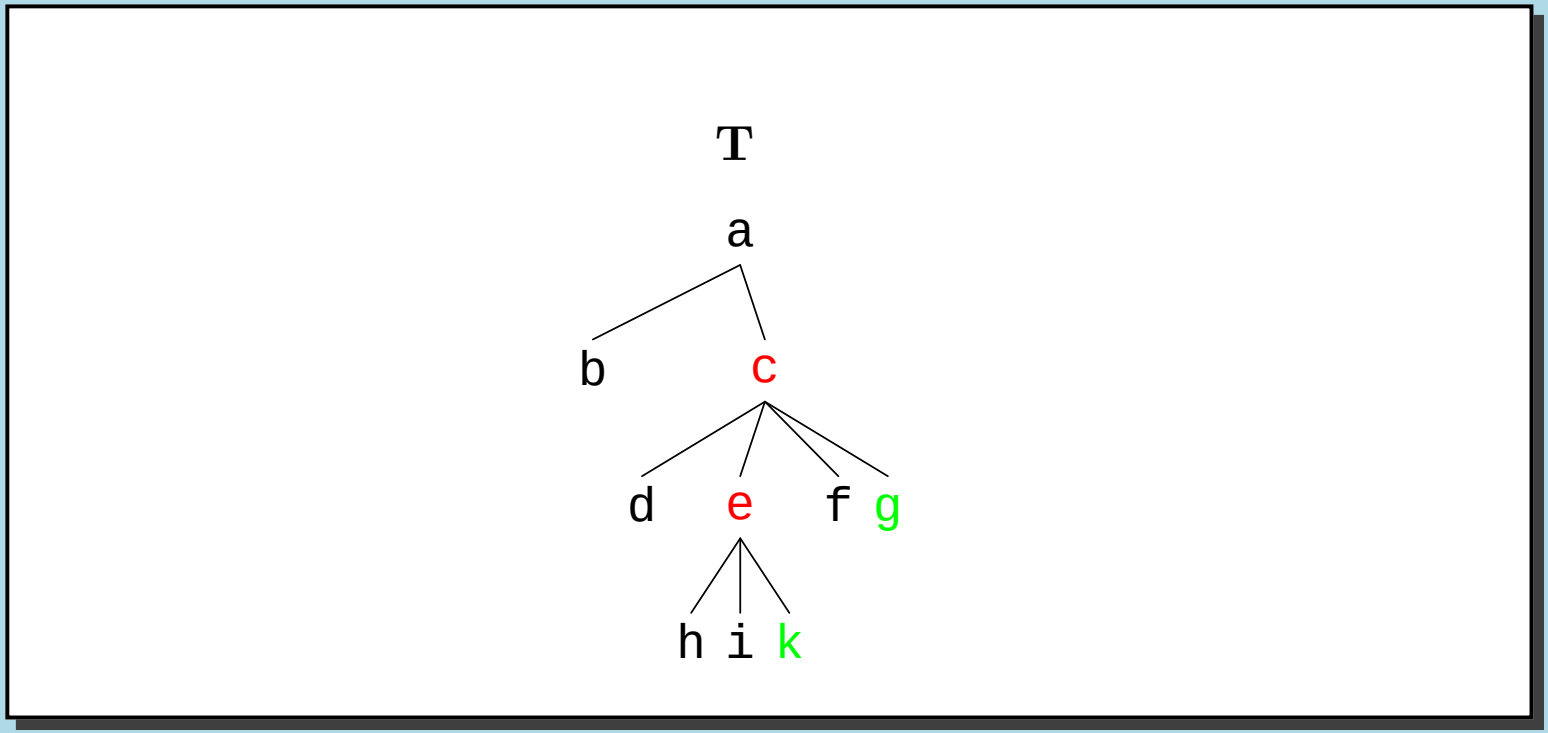
- ⇾ $\Delta^{p,q}(\mathbf{T}_1, \mathbf{T}_2) = 1$ if trees have no pq-grams in common
- ⇾ $\Delta^{p,q}(\mathbf{T}_1, \mathbf{T}_2) = 0$ if trees have the same pq-gram profile

Properties — Sensitivity to Structure Change

☞ **Intuition:** Nodes with structural information ⟶ more significant

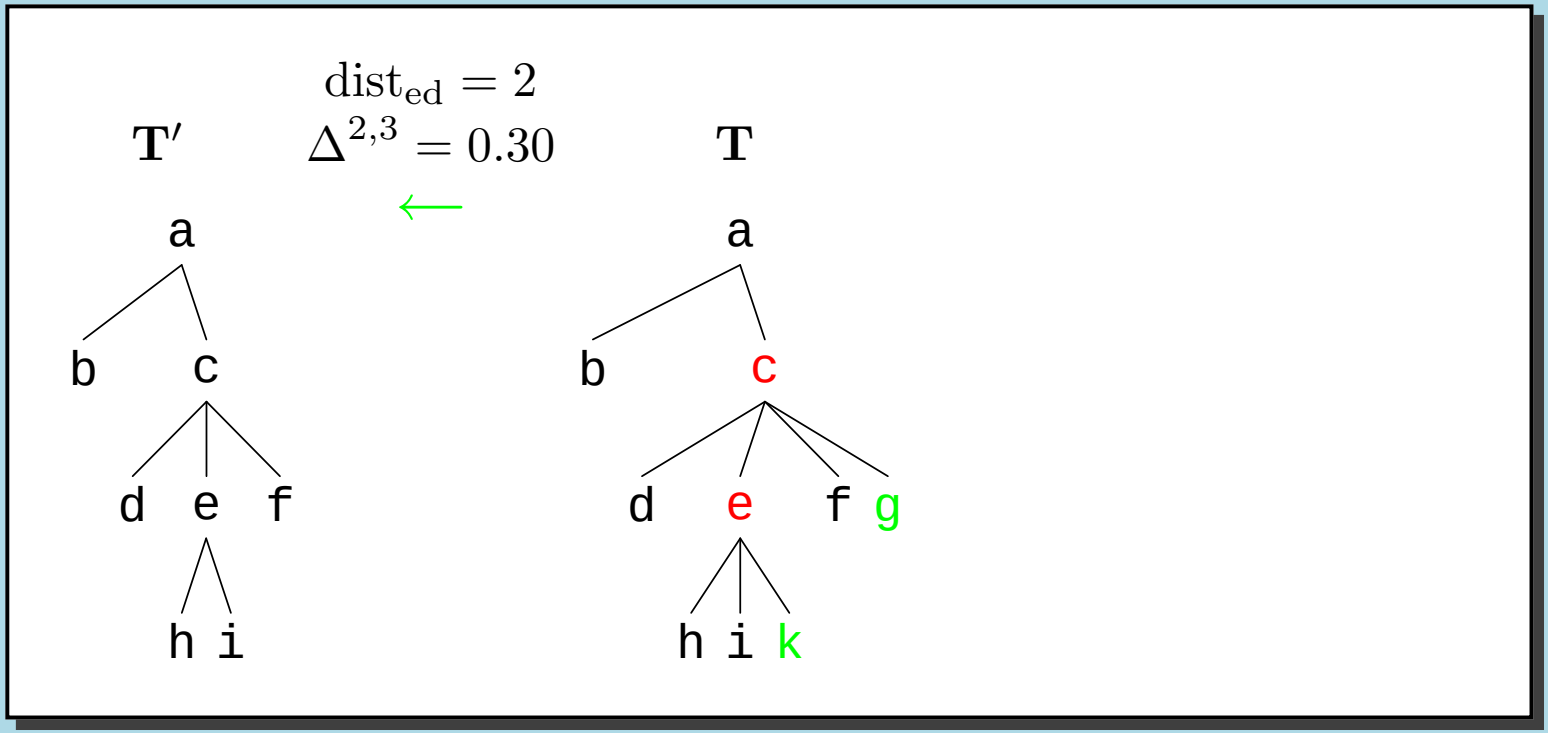
Properties — Sensitivity to Structure Change

☞ **Intuition:** Nodes with structural information ⟶ more significant



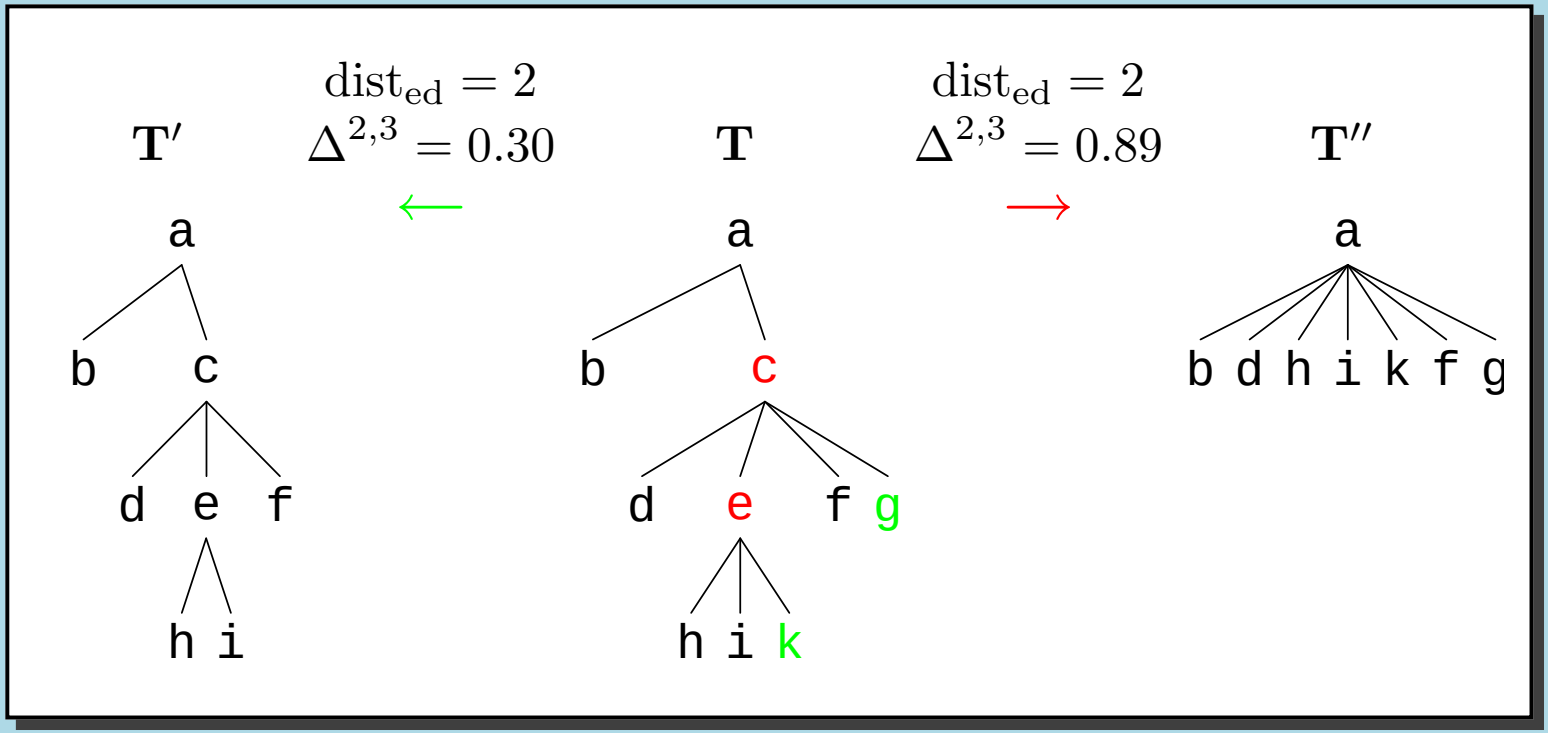
Properties — Sensitivity to Structure Change

☞ **Intuition:** Nodes with structural information → more significant



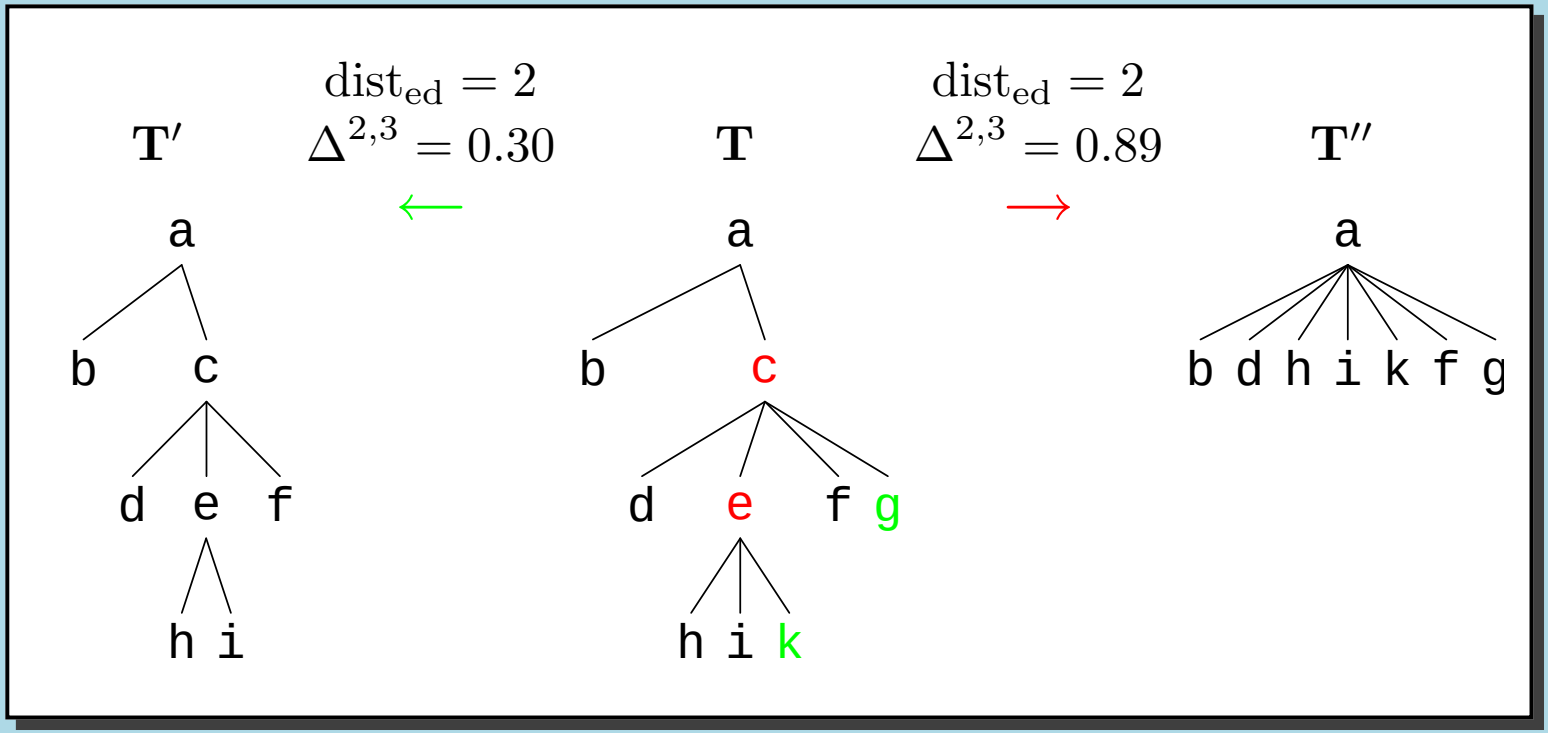
Properties — Sensitivity to Structure Change

☞ **Intuition:** Nodes with structural information → more significant



Properties — Sensitivity to Structure Change

- **Intuition:** Nodes with structural information → more significant
- **Address application:** Mismatch of houses (with subnumbers and apartment numbers) is more significant than mismatch of apartments.



Properties — Sensitive to Structure Change

- ☞ node changes $\Rightarrow$ pq-grams change
- ☞ pq-grams change $\Rightarrow$ distance increases

Properties — Sensitive to Structure Change

- ☞ node changes $\Rightarrow$ pq-grams change
- ☞ pq-grams change $\Rightarrow$ distance increases
- ☞ $\mathrm{cnt}_{pq}(\mathbf{T}, \mathsf{v}) \approx q + f^p$
  - ⇨ **leaf** change: cost depends **only on q**
  - ⇨ **non-leaf** change: **p prevalent**

Properties — Sensitive to Structure Change

- node changes $\Rightarrow$ pq-grams change
- pq-grams change $\Rightarrow$ distance increases
- $\mathrm{cnt}_{pq}(\mathbf{T}, \mathsf{v}) \approx q + f^p$
  - **leaf** change: cost depends **only on q**
  - **non-leaf** change: **p prevalent**

Theorem 2 *For a complete tree* $\mathbf{T}$ *(fanout* f*, depth* d*) the* **number of pq-grams that contain a node** v *of level* l *is:*

$$\mathrm{cnt}_{pq}(\mathbf{T}, \mathsf{v}) = q\,\mathrm{sgn}(l) + \begin{cases} \frac{f^p-1}{f-1}(f+q-1) & \text{if } p \leq d-l \\ \frac{f^{d-l}-1}{f-1}(f+q-1) + f^{d-l} & \text{if } p > d-l. \end{cases}$$

Properties — Sensitive to Structure Change

- node changes $\Rightarrow$ pq-grams change
- pq-grams change $\Rightarrow$ distance increases
- $\mathrm{cnt}_{pq}(\mathbf{T}, \mathsf{v}) \approx q + f^p$
  - **leaf** change: cost depends **only on q**
  - **non-leaf** change: **p prevalent**
  - **p** controls structure sensitivity

Theorem 2 *For a complete tree* $\mathbf{T}$ *(fanout* f*, depth* d*) the* **number of pq-grams that contain a node** v *of level* l *is:*

$$\mathrm{cnt}_{pq}(\mathbf{T}, \mathsf{v}) = q\,\mathrm{sgn}(l) + \begin{cases} \frac{f^p-1}{f-1}(f+q-1) & \text{if } p \leq d-l \\ \frac{f^{d-l}-1}{f-1}(f+q-1) + f^{d-l} & \text{if } p > d-l. \end{cases}$$

Properties — Robust to Local Changes

- **Intuition:** Weight local changes less than distributed changes!

Properties — Robust to Local Changes

- **Intuition:** Weight local changes less than distributed changes!
- **Address application**: missing house with apartment numbers $\longrightarrow$ small difference, but many nodes change

Properties — Robust to Local Changes

- **Intuition:** Weight local changes less than distributed changes!
- **Address application**: missing house with apartment numbers $\longrightarrow$ small difference, but many nodes change
- Local changes $\longrightarrow$ less pq-grams change
  - neighbored nodes “share” pq-grams
  - change counts only once.

Properties — Robust to Local Changes

- **Intuition:** Weight local changes less than distributed changes!
- **Address application**: missing house with apartment numbers $\longrightarrow$ small difference, but many nodes change
- Local changes $\longrightarrow$ less pq-grams change
  - neighbored nodes "share" pq-grams
  - change counts only once.

Theorem 3 *Delete or update all nodes of subtree with l leaves and i non-leaves $\Rightarrow$ only $2l + iq + q - 1$ pq-grams change*

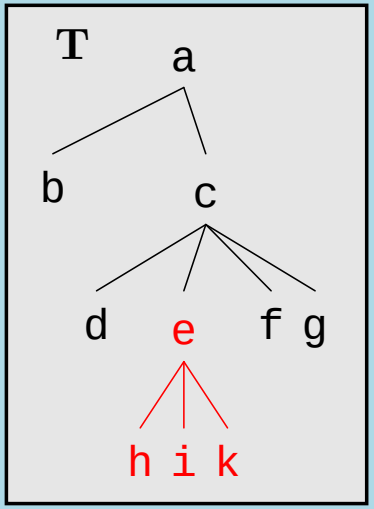
Properties — Robust to Local Changes

- **Intuition:** Weight local changes less than distributed changes!
- **Address application**: missing house with apartment numbers $\longrightarrow$ small difference, but many nodes change
- Local changes $\longrightarrow$ less pq-grams change
  - neighbored nodes "share" pq-grams
  - change counts only once.

Theorem 3 *Delete or update all nodes of subtree with l leaves and i non-leaves $\Rightarrow$ only $2l + iq + q - 1$ pq-grams change*

Example: Delete subtree rooted in e

- e $\longrightarrow$ in 11 pq-grams
- h,i, k $\longrightarrow$ in 4 pq-grams each
- actually changing: 11 pq-grams (vs. $3 \times 4 + 11 = 23$)



Experiments — Sensitivity to Structure Changes

- ☞ Cost for **leaf change** $\longrightarrow$ depends only on q

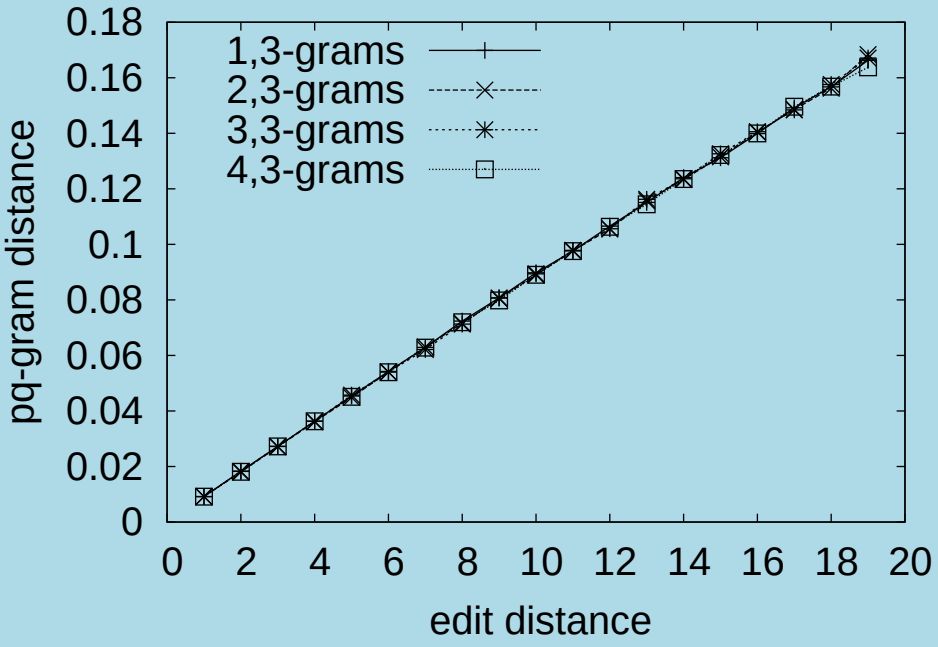
Experiments — Sensitivity to Structure Changes

- Cost for **leaf change** $\longrightarrow$ depends only on q
- **Experiment:**
  - delete leaf nodes
  - measure edit distance

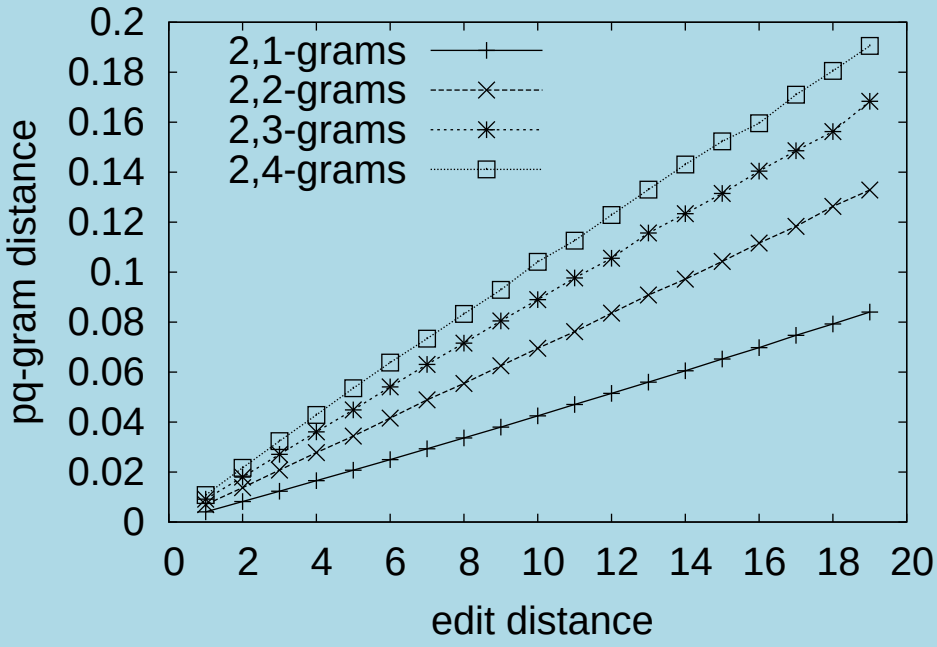
Experiments — Sensitivity to Structure Changes

- Cost for **leaf change** $\longrightarrow$ depends only on q
- **Experiment:**
  - delete leaf nodes
  - measure edit distance

vary p



vary q



(Artifi cial tree with 144 nodes, 102 leaves, fanout 2–6 and depth 6. Average over 100 runs.)

Experiments — Sensitivity to Structure Changes

- ☞ Cost for **non-leaf change** $\longrightarrow$ controlled by p

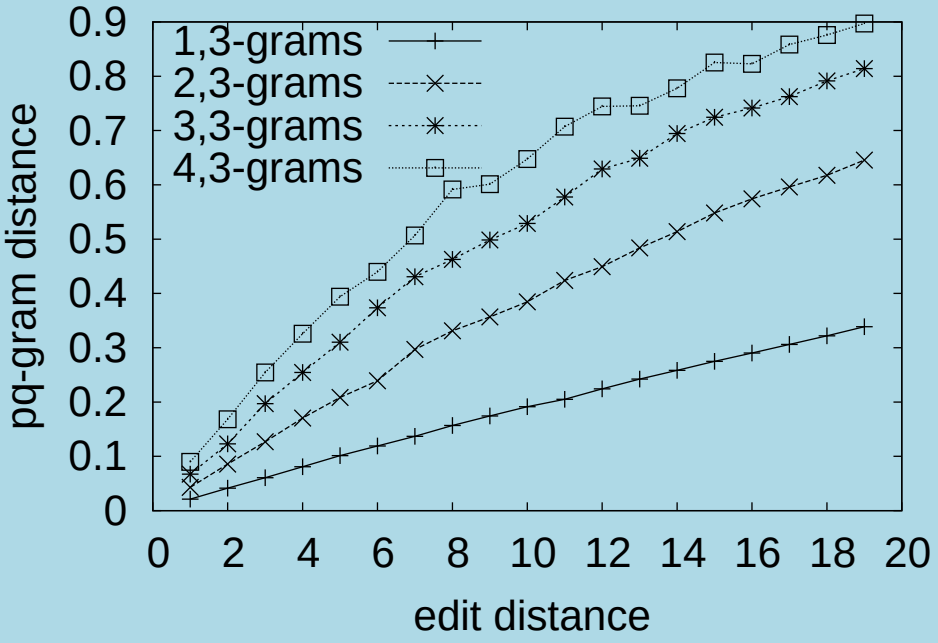
Experiments — Sensitivity to Structure Changes

- Cost for **non-leaf change** $\longrightarrow$ controlled by p
- **Experiment:**
  - delete non-leaf nodes
  - measure edit distance

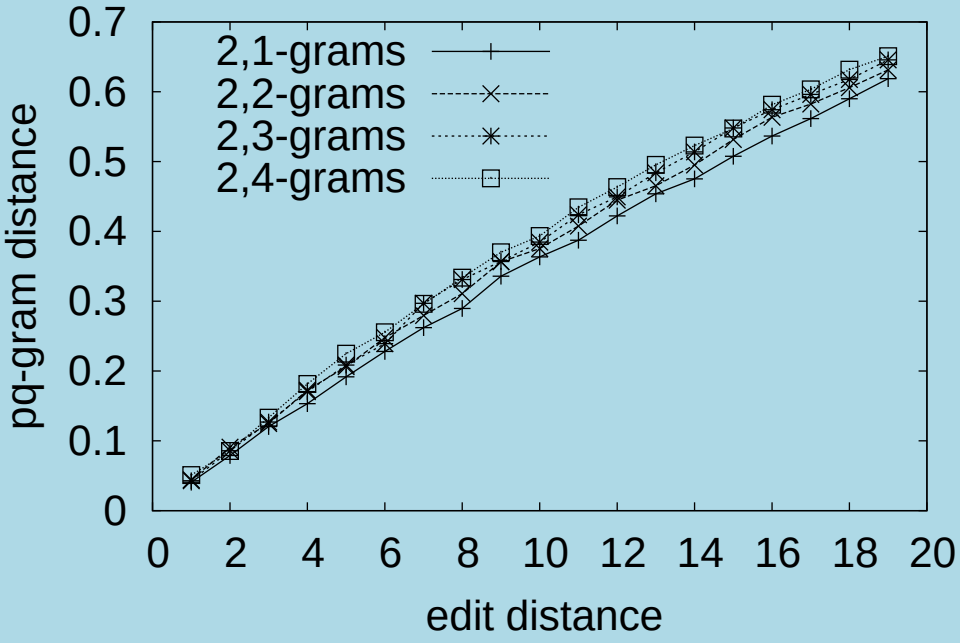
Experiments — Sensitivity to Structure Changes

- Cost for **non-leaf change** → controlled by p
- **Experiment:**
  - delete non-leaf nodes
  - measure edit distance

vary p



vary q



(Artifi cial tree with 144 nodes, 102 leaves, fanout 2–6 and depth 6. Average over 100 runs.)

Experiments — Robustness to Local Changes

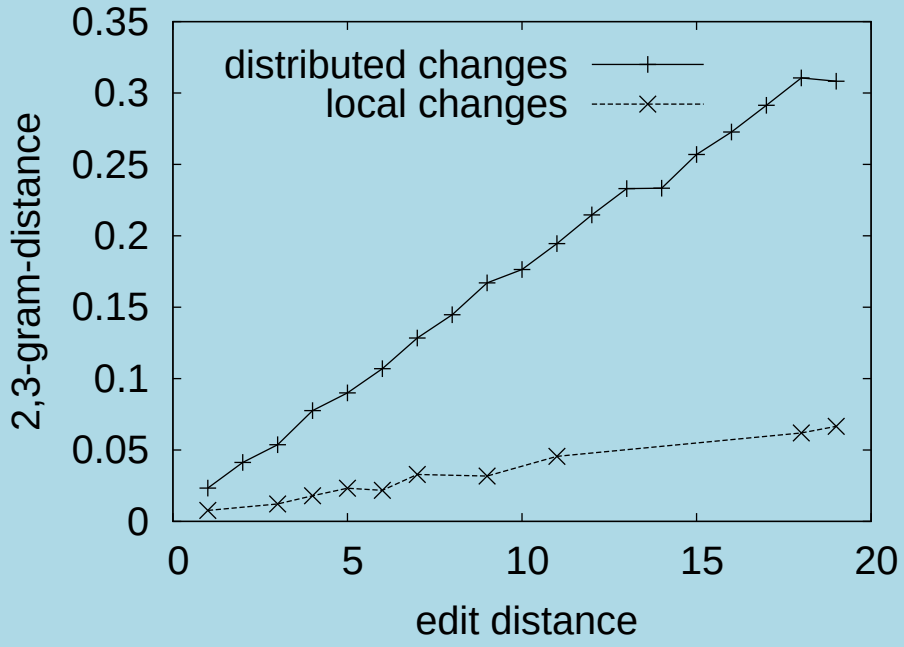
☞ **Subtree deletions** ⟶ **cheaper** than distributed deletions

Experiments — Robustness to Local Changes

- **Subtree deletions** → **cheaper** than distributed deletions
- **Experiment:**
  - delete subtree
  - randomly delete same number of distributed nodes
  - compare edit distance

Experiments — Robustness to Local Changes

- **Subtree deletions** → **cheaper** than distributed deletions
- **Experiment:**
  - delete subtree
  - randomly delete same number of distributed nodes
  - compare edit distance



(Artifi cial tree with 144 nodes, 102 leaves, fanout 2–6 and depth 6. Average over 100 runs.)

Experiments — Scalability to Large Trees

- ☞ pq-gram distance $\longrightarrow$ **scalable** to large trees
- ☞ compare with edit distance

[a] implementation provided by Zhang and Shasha http://www.cs.nyu.edu/cs/faculty/shasha/papers/tre

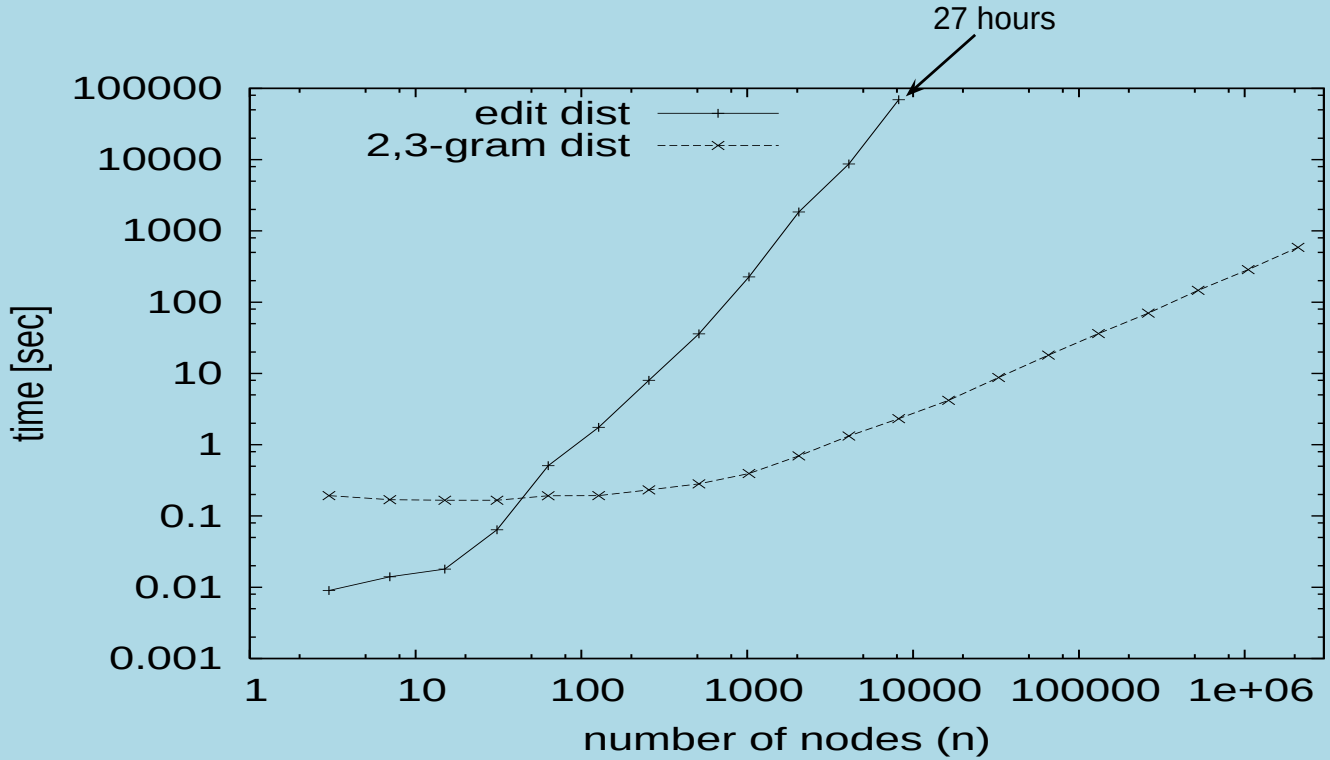
Experiments — Scalability to Large Trees

- pq-gram distance $\longrightarrow$ **scalable** to large trees
- compare with edit distance
- **Experiment:** For pair of trees
  - compute tree edit distance[a] and pq-gram distance
  - vary tree size: up 2×10^6 nodes
  - measure wall clock time

[a] implementation provided by Zhang and Shasha http://www.cs.nyu.edu/cs/faculty/shasha/papers/tre

Experiments — Scalability to Large Trees

- pq-gram distance $\longrightarrow$ **scalable** to large trees
- compare with edit distance
- **Experiment:** For pair of trees
  - compute tree edit distance[a] and pq-gram distance
  - vary tree size: up 2×10^6 nodes
  - measure wall clock time



[a] implementation provided by Zhang and Shasha http://www.cs.nyu.edu/cs/faculty/shasha/papers/tre

Experiments — Influence of p and q on Scalability

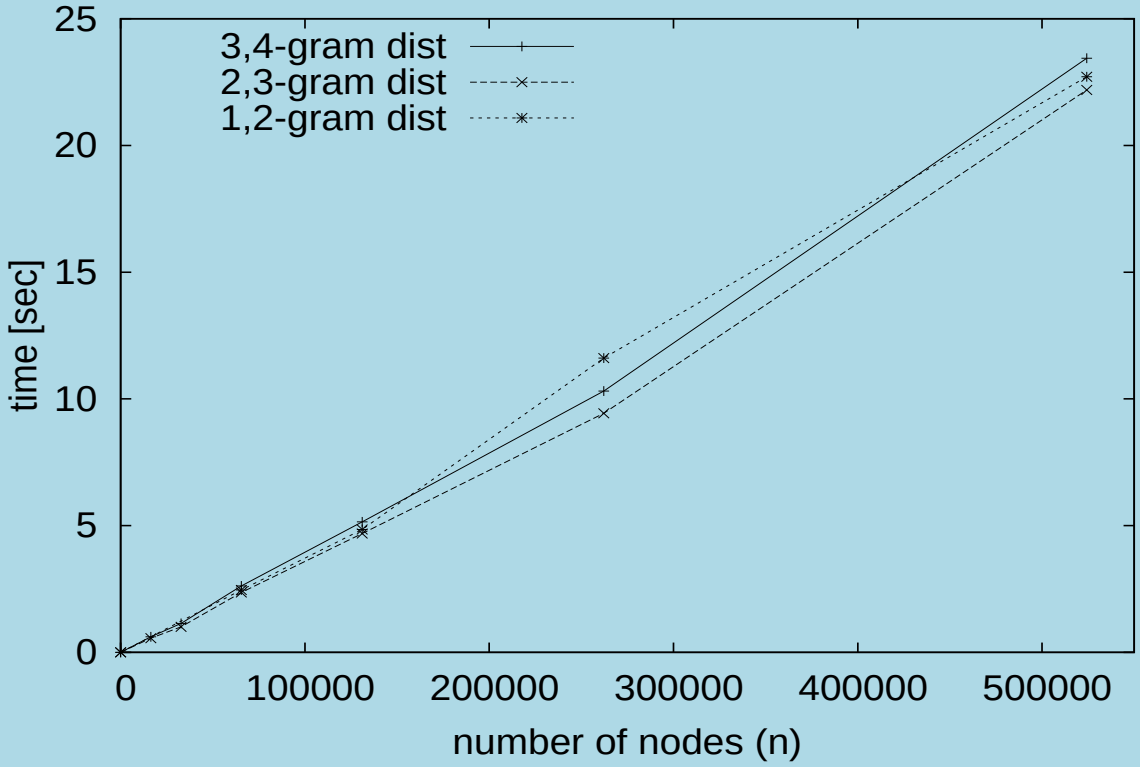
- ☞ Scalability independent of p and q.

Experiments — Influence of p and q on Scalability

- Scalability independent of p and q.
- **Experiment:** For pair of trees
  - compute pq-gram distance for varying p and q
  - vary tree size: up 10^6 nodes
  - measure wall clock time

Experiments — Influence of p and q on Scalability

- Scalability independent of p and q.
- **Experiment:** For pair of trees
  - compute pq-gram distance for varying p and q
  - vary tree size: up 10^6 nodes
  - measure wall clock time



Experiments — Effectiveness for Real World Dataset

- ☞ pq-gram distance $\longrightarrow$ **effective approximation** of tree edit distance
- ☞ test on real world data (**address databases**)

Experiments — Effectiveness for Real World Dataset

- pq-gram distance $\longrightarrow$ **effective approximation** of tree edit distance
- test on real world data (**address databases**)
- **Experiment:** For two sets of address trees
  - find matches (closest tree of other set)
  - use different distance functions
  - count correct matches

Experiments — Effectiveness for Real World Dataset

- pq-gram distance $\longrightarrow$ **effective approximation** of tree edit distance
- test on real world data (**address databases**)
- **Experiment:** For two sets of address trees
  - find matches (closest tree of other set)
  - use different distance functions
  - count correct matches

	accuracy	correct	false pos.	runtime
edit dist	82.7%	248	9	187,538s
1,2-grams	78.3%	235	5	181s
2,3-grams	77.3%	232	4	204s
3,2-grams	79.3%	238	2	180s
tree-embedding	69.0%	207	8	313s
buttom-up	50.0%	150	12	237s

Experiments — Tree-Embedding vs. pq-Grams

tree edit distance embedding	pq-grams
☞ variable shapes	☞ fixed shape

Experiments — Tree-Embedding vs. pq-Grams

tree edit distance embedding	pq-grams
☞ variable shapes	☞ fixed shape
☞ elements can be ⇨ single node (no structure) ⇨ chains (only vertical structure) ⇨ contiguous leaves (only horizontal structure) ⇨ subtrees with vertical and horizontal structure	☞ horizontal **and** vertical structure

Typical address tree (nearly complete tree):

Phase	0	1	2	3	4	5	6	tot.	tree-embed	pq-gram
single nodes	29	8	4	2	1	-	-	44	65%	0%
chains	-	1	1	-	-	-	-	2	3%	0%
cont. leaves	-	7	2	-	-	-	-	9	13%	0%
subtrees	-	-	3	4	3	2	1	13	19%	100%

Experiments — Tree-Embedding vs. pq-Grams

tree edit distance embedding	pq-grams
☞ variable shapes	☞ fixed shape
☞ elements can be ➪ single node (no structure) ➪ chains (only vertical structure) ➪ contiguous leaves (only horizontal structure) ➪ subtrees with vertical and horizontal structure	☞ horizontal **and** vertical structure
☞ guarantees with respect to tree edit distance	☞ emphasizes structure

Typical address tree (nearly complete tree):

Phase	0	1	2	3	4	5	6	tot.	tree-embed	pq-gram
single nodes	29	8	4	2	1	-	-	44	65%	0%
chains	-	1	1	-	-	-	-	2	3%	0%
cont. leaves	-	7	2	-	-	-	-	9	13%	0%
subtrees	-	-	3	4	3	2	1	13	19%	100%

Conclusion and Future Work

- ☞ pq-gram distance

Conclusion and Future Work

- ☞ pq-gram distance
  - ⇨ scalable to large trees

Conclusion and Future Work

- pq-gram distance
  - scalable to large trees
  - emphasizes structure

Conclusion and Future Work

- pq-gram distance
  - scalable to large trees
  - emphasizes structure
  - robust to local change

Conclusion and Future Work

- pq-gram distance
  - scalable to large trees
  - emphasizes structure
  - robust to local change
  - effective approximation of tree edit distance

Conclusion and Future Work

- pq-gram distance
  - scalable to large trees
  - emphasizes structure
  - robust to local change
  - effective approximation of tree edit distance
- Ongoing and future work:

Conclusion and Future Work

- pq-gram distance
  - scalable to large trees
  - emphasizes structure
  - robust to local change
  - effective approximation of tree edit distance
- Ongoing and future work:
  - strict bounds for pq-gram approximations

Conclusion and Future Work

- pq-gram distance
  - scalable to large trees
  - emphasizes structure
  - robust to local change
  - effective approximation of tree edit distance
- Ongoing and future work:
  - strict bounds for pq-gram approximations
  - clustering of XML data

Conclusion and Future Work

- pq-gram distance
  - scalable to large trees
  - emphasizes structure
  - robust to local change
  - effective approximation of tree edit distance
- Ongoing and future work:
  - strict bounds for pq-gram approximations
  - clustering of XML data
  - incremental updates of pq-gram profiles

References

[Chawathe and Garcia-Molina, 1997] Chawathe, S. S. and Garcia-Molina, H. (1997). Meaningful change detection in structured data. In *Proceedings of the ACM SIGMOD International Conference on Management of Data*, pages 26–37, Tucson, Arizona, United States. ACM Press.

[Chawathe et al., 1996] Chawathe, S. S., Rajaraman, A., Garcia-Molina, H., and Widom, J. (1996). Change detection in hierarchically structured information. In *Proceedings of the ACM SIGMOD International Conference on Management of Data*, pages 493–504, Montreal, Quebec, Canada. ACM Press.

[Garofalakis and Kumar, 2003] Garofalakis, M. and Kumar, A. (2003). Correlating XML data streams using tree-edit distance embeddings. In *Proceedings of the Twenty-Second ACM SIGACT-SIGMOD-SIGART Symposium on Principles of Database Systems (PODS 2003)*, pages 143–154, San Diego, California. ACM Press.

[Garofalakis and Kumar, 2005] Garofalakis, M. and Kumar, A. (2005). XML stream processing using tree-edit distance embeddings. *ACM Transactions on Database Systems*, 30(1):279–332.

[Jiang et al., 1995] Jiang, T., Wang, L., and Zhang, K. (1995). Alignment of trees—an alternative to tree edit. *Theoretical Computer Science*, 143(1):137–148.

[Klein, 1998] Klein, P. N. (1998). Computing the edit-distance between unrooted ordered trees. In *Proceedings of the 6th European Symposium on Algorithms*, volume 1461 of *Lecture Notes in Computer Science*, pages 91–102, Venice, Italy. Springer.

[Lee et al., 2004] Lee, K.-H., Choy, Y.-C., and Cho, S.-B. (2004). An efficient algorithm to compute differences between structured documents. *IEEE Transactions on Knowledge and Data Engineering*, 16(8):965–979.

[Navarro, 2001] Navarro, G. (2001). A guided tour to approximate string matching. *ACM Computing Surveys*, 33(1):31–88.

[Selkow, 1977] Selkow, S. M. (1977). The tree-to-tree editing problem. *Information Processing Letters*, 6(6):184–186.

[Tanaka and Tanaka, 1988] Tanaka, E. and Tanaka, K. (1988). The tree-to-tree editing problem. *Int. Journal of Pattern Recognition and Artificial Intelligence (IJPRAI)*, 2(2):221–240.

[Ukkonen, 1992] Ukkonen, E. (1992). Approximate string-matching with q-grams and maximal matches. *Theoretical Computer Science*, 92(1):191–211.

[Valiente, 2001] Valiente, G. (2001). An efficient bottom-up distance between trees. In *Proceedings of the 8th Symposium on String Processing and Information Retrieval*, pages 212–219, Laguna de San Rafael, Chile. IEEE Computer Science Press.

[Yang, 1991] Yang, W. (1991). Identifying syntactic differences between two programs. *Software—Practice & Experience*, 21(7):739–755.

[Zhang and Shasha, 1989] Zhang, K. and Shasha, D. (1989). Simple fast algorithms for the editing distance between trees and related problems. *SIAM Journal on Computing*, 18(6):1245–1262.