Charu C. Aggarwal
T J Watson Research Center
IBM Corporation
Hawthorne, NY
USA

On k-anonymity and the curse of dimensionality

Introduction

- An important method for privacy preserving data mining is that of *anonymization*.
- In anonymization, a record is released only if it is indistinguishable from a pre-defined number of other entities in the data.
- We examine the anonymization problem from the perspective of inference attacks over all possible combinations of attributes.

Public Information

- In k-anonymity, the premise is that public information can be combined with the attribute values of anonymized records in order to identify the identities of records.

- Such attributes which are matched with public records are referred to as *quasi-identifiers*.

- For example, a commercial database containing birthdates, gender and zip-codes can be matched with voter registration lists in order to identify the individuals precisely.

Example

- Consider the following 2-dimensional records on (Age, Salary) $= (26, 94000)$ and $(29, 97000)$.

- Then, if age is generalized to the range 25-30, and if salary is generalized to the range 90000-100000, then the two records cannot be distinguished from one another.

- In k-anonymity, we would like to provide the guarantee that each record cannot be distinguished from at least $(k - 1)$ other records.

- In such a case, even public information cannot be used to make inferences.

The k-anonymity method

- The method of k-anonymity typically uses the techniques of generalization and suppression.

- Individual attribute values and records can be suppressed.

- Attributes can be partially generalized to a range (retains more information than complete suppression).

- The generalization and suppression process is performed so as to create at least k indistinguishable records.

The condensation method

- An alternative to generalization and suppression methods is the condensation technique.

- In the condensation method, clustering techniques are used in order to construct indistinguishable groups of k records.

- The statistical characteristics of these clusters are used to generate pseudo-data which is used for data mining purposes.

- There are some advantages in the use of pseudo-data, since it does not require any modification of the underlying data representation as in a generalization approach.

High Dimensional Case

- Typical anonymization approaches assume that only a small number of fields which are available from public data are used as quasi-identifiers.

- These methods typically use generalizations on domain-specific hierarchies of these small number of fields.

- In many practical applications, large numbers of attributes may be known to particular groups of individuals.

- Larger number of attributes make the problem more challenging for the privacy preservation process.

Challenges

- The problem of finding optimal k-anonymization is NP-hard.
- This computational problem is however secondary, if the data cannot be anonymized effectively.
- We show that in high dimensionality, it becomes more difficult to perform the generalizations on partial ranges in a meaningful way.

Anonymization and Locality

- All anonymization techniques depend upon some notion of spatial locality in order to perform the privacy preservation.

- Generalization based locality is defined in terms of ranges of attributes.

- Locality is also defined in the form of a distance function in condensation approaches.

- Therefore, the behavior of the anonymization approach will depend upon the behavior of the distance function with increasing dimensionality.

Locality Behavior in High Dimensionality

- It has been argued that under certain reasonable assumptions on the data distribution, the distances of the nearest and farthest neighbors to a given target in high dimensional space is almost the same for a variety of data distributions and distance functions (Beyer et al).

- In such a case, the concept of spatial locality becomes ill defined.

- Privacy preservation by anonymization becomes impractical in very high dimensional cases, since it leads to an unacceptable level of information loss.

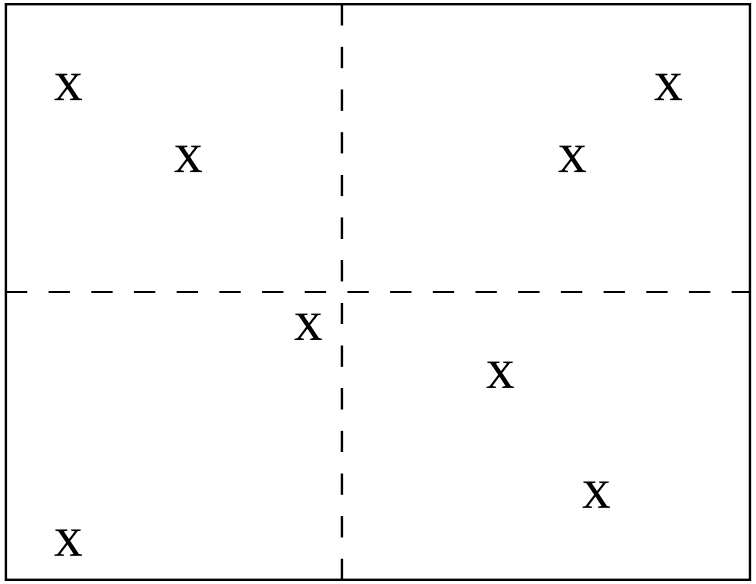
Notations and Definitions

Notation	Definition
d	Dimensionality of the data space
N	Number of data points
$\mathcal{F}$	1-dimensional data distribution in $(0, 1)$
X_d	Data point from $\mathcal{F}^d$ with each coord. drawn from $\mathcal{F}$
$dist_d^k(x, y)$	Distance between $(x^1, \ldots x^d)$ and $(y^1, \ldots y^d)$ using L_k metric $= \sum_{i=1}^{d}[(x_1^i - x_2^i)^k]^{1/k}$
$\\| \cdot \\|_k$	Distance of a vector to the origin $(0, \ldots, 0)$ using the function $dist_d^k(\cdot, \cdot)$
$E[X]$, $var[X]$	Expected value and variance of a random variable X
$Y_d \rightarrow_p c$	A sequence of vectors $Y_1, \ldots, Y_d$ converges in probability to a constant vector c if: $\forall \epsilon > 0 \; lim_{d \rightarrow \infty} P[dist_d(Y_d, c) \leq \epsilon] = 1$

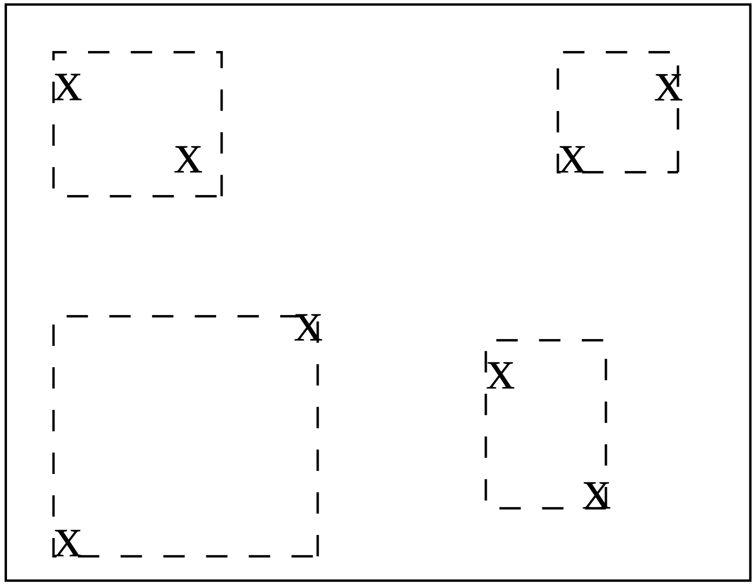
Range based generalization

- In range based generalization, we generalize the attribute values to a range such that at least k records can be found in the generalized grid cell.
- In the high dimensional case, most grid cells are empty.
- But what about the non-empty grid cells?
- How is the data distributed among the non-empty grid cells?

Illustration



(a)



(b)

Attribute Generalization

- Let us consider the axis-parallel generalization approach, in which individual attribute values are replaced by a randomly chosen interval from which they are drawn.

- In order to analyze the behavior of anonymization approaches with increasing dimensionality, we consider the case of data in which individual dimensions are independent and identically distributed.

- The resulting bounds provide insight into the behavior of the anonymization process with increasing *implicit* dimensionality.

Assumption

- For a data point $\overline{X_d}$ to maintain k-anonymity, its bounding box must contain *at least* $(k-1)$ other points.

- First, we will consider the case when the generalization of each point uses a maximum fraction f of the data points along each of the d partially specified dimensions.

- It is interesting to compute the conditional probability of k-anonymity in a randomly chosen grid cell, given that it is non-empty.

- Provides intuition into the probability of k-anonymity in a multi-dimensional partitioning.

Result (Lemma 1)

- Let $\mathcal{D}$ be a set of N points drawn from the d-dimensional distribution $\mathcal{F}^d$ in which individual dimensions are independently distributed. Consider a randomly chosen grid cell, such that each partially masked dimension contains a fraction f of the total data points in the specified range. Then, the probability P^q of exactly q points in the cell is given by $\binom{N}{q} \cdot f^{q \cdot d} \cdot (1 - f^d)^{(N-q)}$.

- Simple binomial distribution with parameter f^d.

Result (Lemma 2)

- Let B_k be the event that the set of partially masked ranges contains at least k data points. Then the following result for the conditional probability $P(B_k|B_1)$ holds true:

$$P(B_k|B_1) = \frac{\sum_{q=k}^{N} \binom{N}{q} \cdot f^{q\cdot d} \cdot (1 - f^d)^{(N-q)}}{\sum_{q=1}^{N} \binom{N}{q} \cdot f^{q\cdot d} \cdot (1 - f^d)^{(N-q)}} \tag{1}$$

- $P(B_k|B_1) = P(B_k \cap B_1)/P(B_1) = P(B_k)/P(B_1)$

- **Observation:** $P(B_k|B_1) \leq P(B_2|B_1)$

- **Observation:** $P(B_2|B_1) = \frac{1-N\cdot f^d\cdot(1-f^d)^{(N-1)}-(1-f^d)^N}{1-(1-f^d)^N}$

Result

- Substitute $x = f^d$ and use L'Hopital's rule

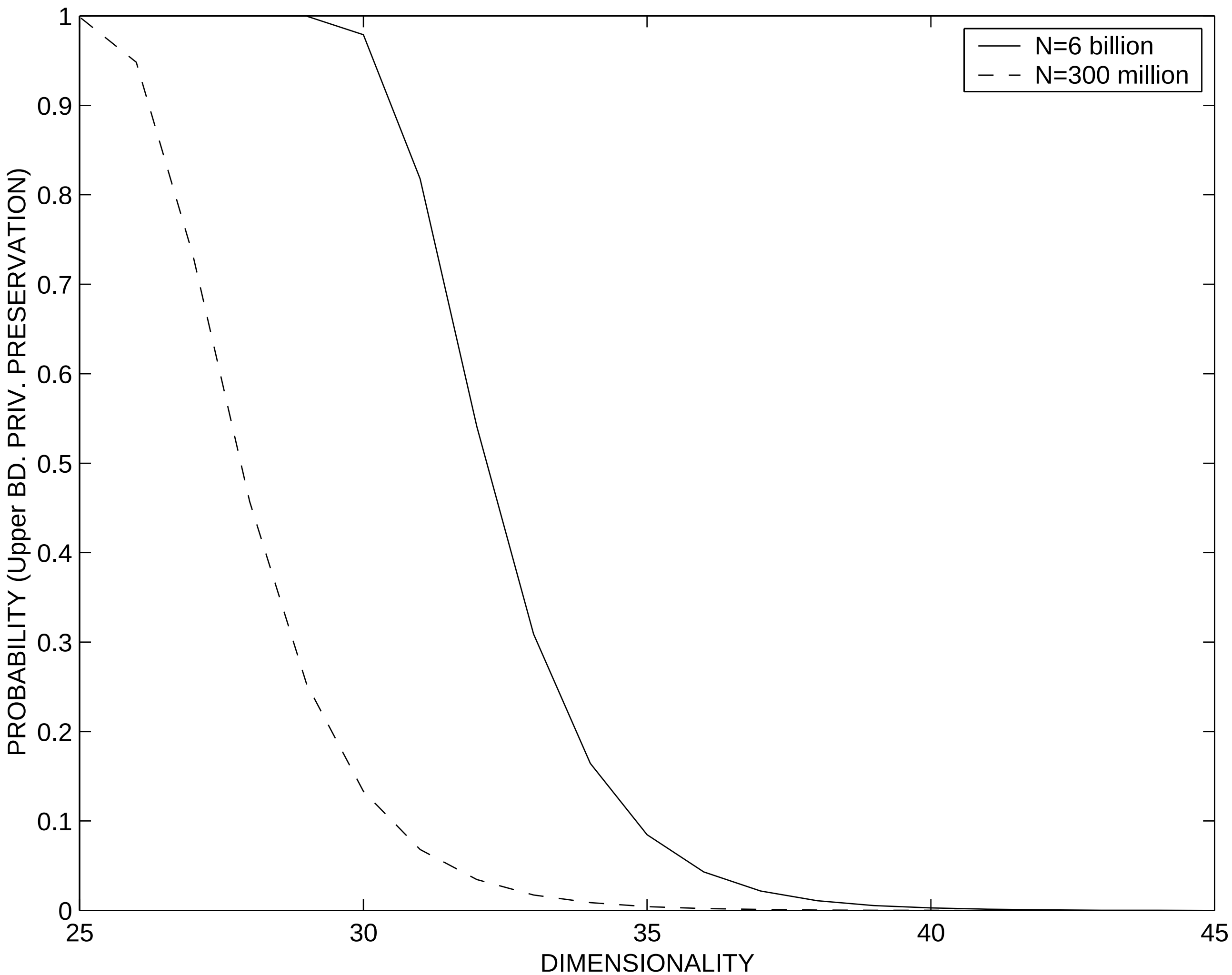
$$P(B_2|B_1) = 1 - \lim_{x \to 0} \frac{N \cdot (1-x)^{(N-1)} - N \cdot x \cdot (1-x)^{(N-2)}}{N \cdot (1-x)^{(N-1)}}$$

- Expression tends to zero as $d \to \infty$

- The limiting probability for achieving k-anonymity in a non-empty set of masked ranges containing a fraction $f < 1$ of the data points is zero. In other words, we have:

$$\lim_{d \to \infty} P(B_k|B_1) = 0 \qquad (2)$$

Probability of 2-anonymity with increasing dimensionality (f=0.5)



The Condensation Approach

- Previous analysis is for range generalization.

- Methods such as condensation use multi-group cluster formation of the records.

- In the following, we will find a lower bound on the information loss for achieving 2-anonymity using any kind of optimized group formation.

Information Loss

- We assume that a set S of k data points are merged together in one group for the purpose of condensation.

- Let $M(S)$ be the maximum euclidian distance between any pair of data points in this group from database $\mathcal{D}$.

- We note that larger values of $M(S)$ represent a greater loss of information, since the points within a group cannot be distinguished for the purposes of data mining.

- We define the *relative condensation loss* $\mathcal{L}(S)$ for that group of k entities as follows:

$$\mathcal{L}(S) = M(S)/M(\mathcal{D}) \tag{3}$$

Observations

- A value of $\mathcal{L}(S)$ which is close to one implies that most of the distinguishing information is lost as a result of the privacy preservation process.

- In the following analysis, we will show how the value of $\mathcal{L}(S)$ is affected by the dimensionality d.

Assumptions

- We first analyze the behavior of a uniform distribution of $N = 3$ data points, and deal with the particular case of 2-anonymity.

- For ease in analysis, we will assume that one of these 3 points is the origin O_d, and the remaining two points are A_d and B_d which are uniformly distributed in the data cube.

- We also assume that the closest of the two points A_d and B_d need to be merged with O_d in order to preserve 2-anonymity of O_d. We establish some convergence results.

- We will also generalize the results to the case of $N = n$ data points.

Lemma

- Let $\mathcal{F}^d$ be uniform distribution of $N = 2$ points. Let us assume that the closest of the 2 points to O_d is merged with O_d to preserve 2-anonymity of the underlying data. Let q_d be the Euclidean distance of O_d to the merged point, and let r_d be the distance of O_d to the remaining point. Then, we have: $\lim_{d \to \infty} E[r_d - q_d] = C$, where C is some constant.

- Multiply numerator and denominator by $r_d + q_d$ and proceed.

Result

- Let $A_d = (P_1 \ldots P_d)$ and $B_d = (Q_1 \ldots Q_d)$ with P_i and Q_i being drawn from $\mathcal{F}$.

- Let $PA_d = \{\sum_{i=1}^{d}(P_i)^2\}^{1/2}$ be the distance of A_d to the origin O_d, and $PB_d = \{\sum_{i=1}^{d}(Q_i)^2\}^{1/2}$ the distance of B_d from O_d.

- $|PA_d - PB_d| = \frac{|(PA_d)^2 - (PB_d)^2|}{(PA_d)+(PB_d)}$

- Analyze the convergence behavior of the numerator and denominator separately in conjunction with Slutsky's results.

Generalization to N points

- Let $\mathcal{F}^d$ be uniform distribution of $N = n$ points. Let us assume that the closest of the n points is merged with O_d to preserve 2-anonymity. Let q_d be the Euclidean distance of O_d to the merged point, and let r_d be the distance of the furthest point from O_d. Then, we have: $C''' \leq \lim_{d \to \infty} E\left[r_d - q_d\right] \leq (n-1) \cdot C'''$, where C''' is some constant.

- Direct extension of previous result.

Lemma

- Let $\mathcal{F}^d$ be uniform distribution of $N = n$ points. Let us assume that the closest of the n points is merged with O_d to preserve 2-anonymity. Let q_d be the Euclidean distance of O_d to the merged point, and let r_d be the distance of the furthest point from O_d. Then, we have: $\lim_{d\to\infty} E\left[\frac{r_d - q_d}{r_d}\right] = 0$, where C''' is some constant.

- This result can be proved by showing that $r_d \to_p \sqrt{d}$.

- Note that the distance of each point to the origin in d-dimensional space increases at this rate.

Information Loss for High Dimensional Case

- We note that the information loss $M(S)/M(\mathcal{D})$ for 2-anonymity can be expressed as $1 - E\left[\frac{r_d - q_d}{r_d}\right]$.

- This expression converges to 1 in the limiting case as $d \rightarrow \infty$.

- We are approximating $M(\mathcal{D})$ to r_d since the origin of the cube is probabilistically expected to be one of extreme corners among the maximum distance pair in the database.

Result

- Bounds for 2-anonymity are lower bounds on the general case of k-anonymity.

- For any set S of data points to achieve k-anonymity, the information loss on the set of points S must satisfy:

$$\lim_{d \to \infty} E[M(S)/M(\mathcal{D})] = 1 \tag{4}$$

Experimental Results

- The synthetic data sets were generated as Gaussian clusters with randomly distributed centers in the unit cube.

- The radius along each dimension of each of the clusters was a random variable with a mean of 0.075 and standard deviation of 0.025.

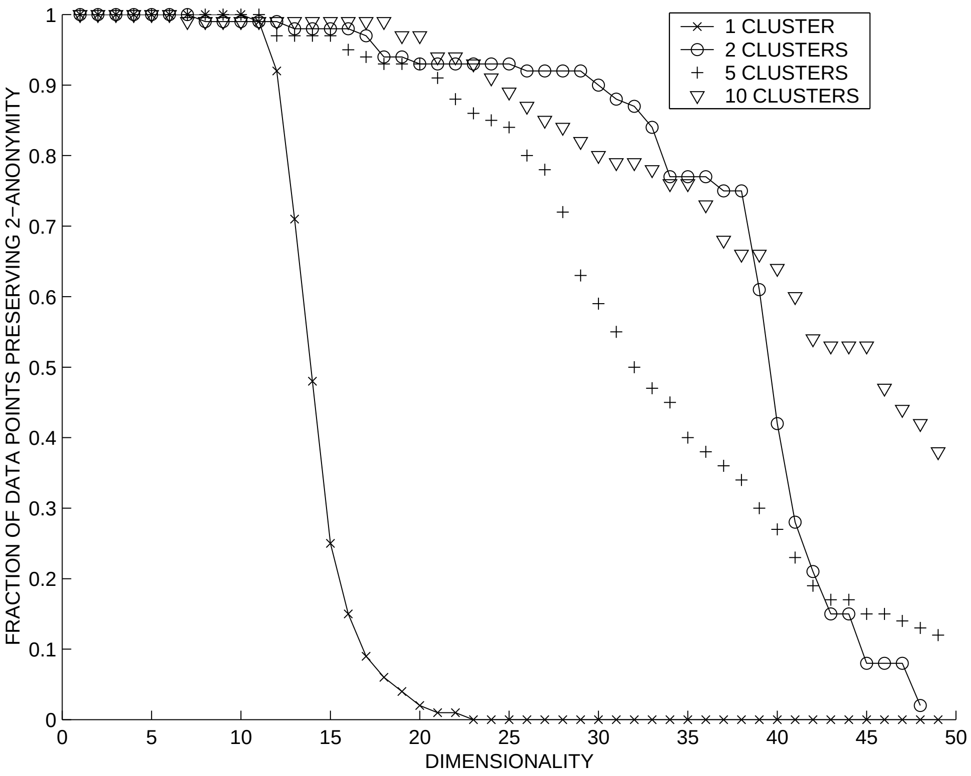
- Thus, a given cluster could be elongated differently along different dimensions by varying the corresponding standard deviation.

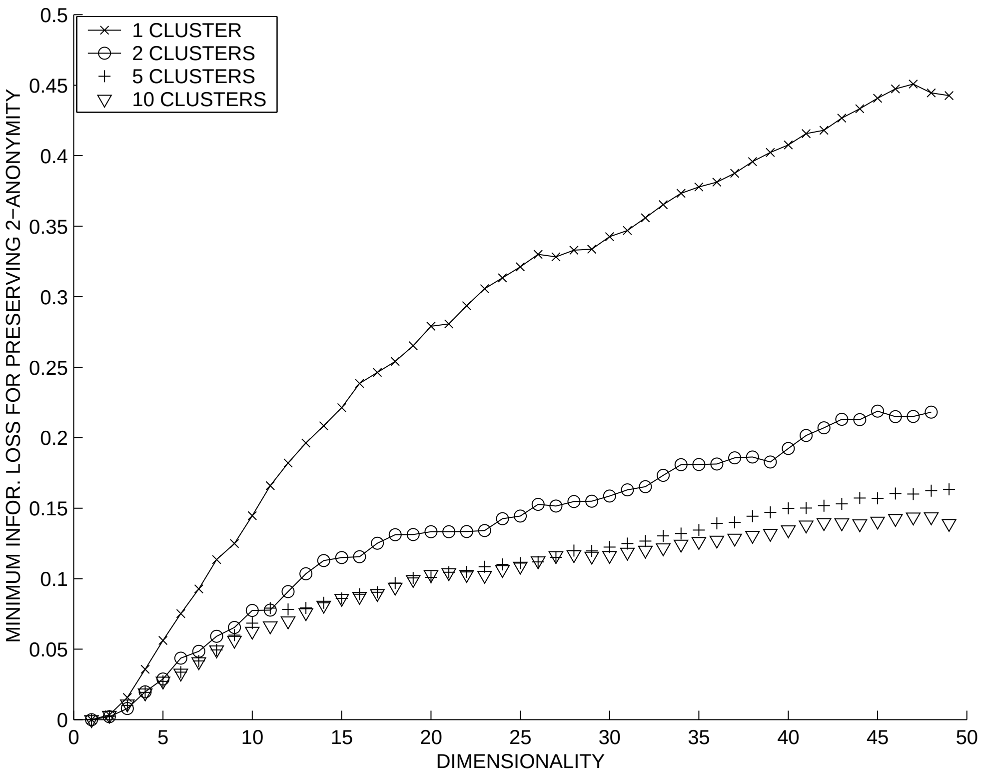
- Each data set was generated with $N = 10000$ data points in a total of 50 dimensions.

Market Basket Data Sets

- We also tested the anonymization behavior with a number of market basket data sets.
- These data sets were generated using the data generator , except that the dimensionality was reduced to only 100 items.
- In order to anonymize the data, each customer who bought an item was masked by also including other random customers as buyers of that item.
- Thus, this experiment to useful to illustrate the effect of our technique on categorical data sets.
- As a result, for each item, the masked data showed that 50% of the customers had bought it, and the other 50% had not bought it.

Experimental Results





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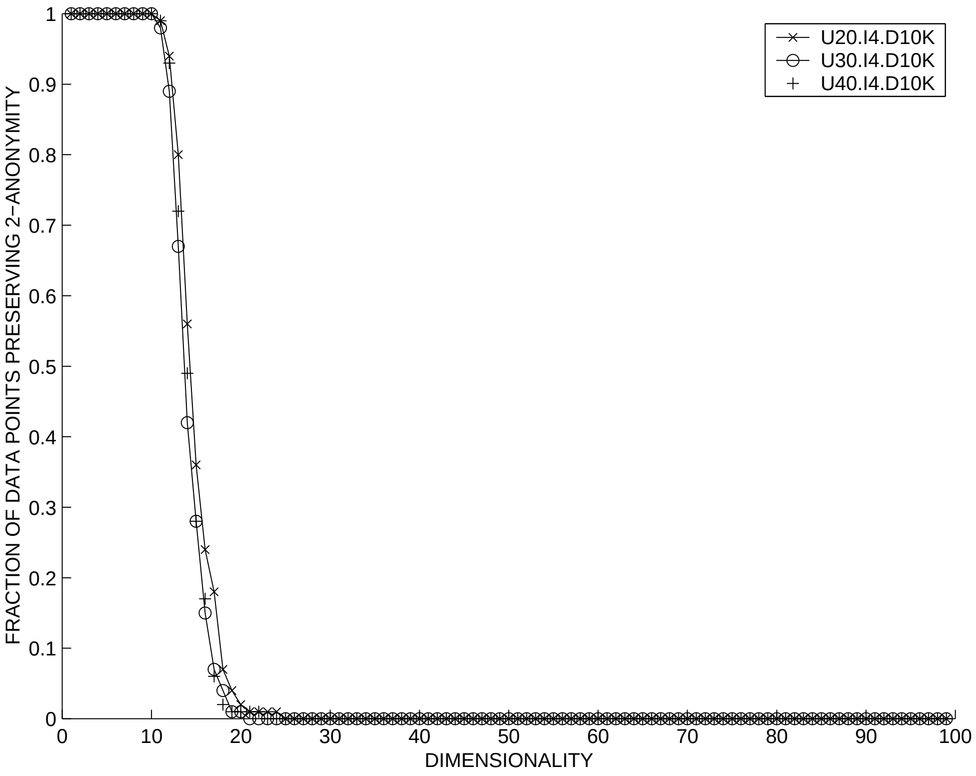

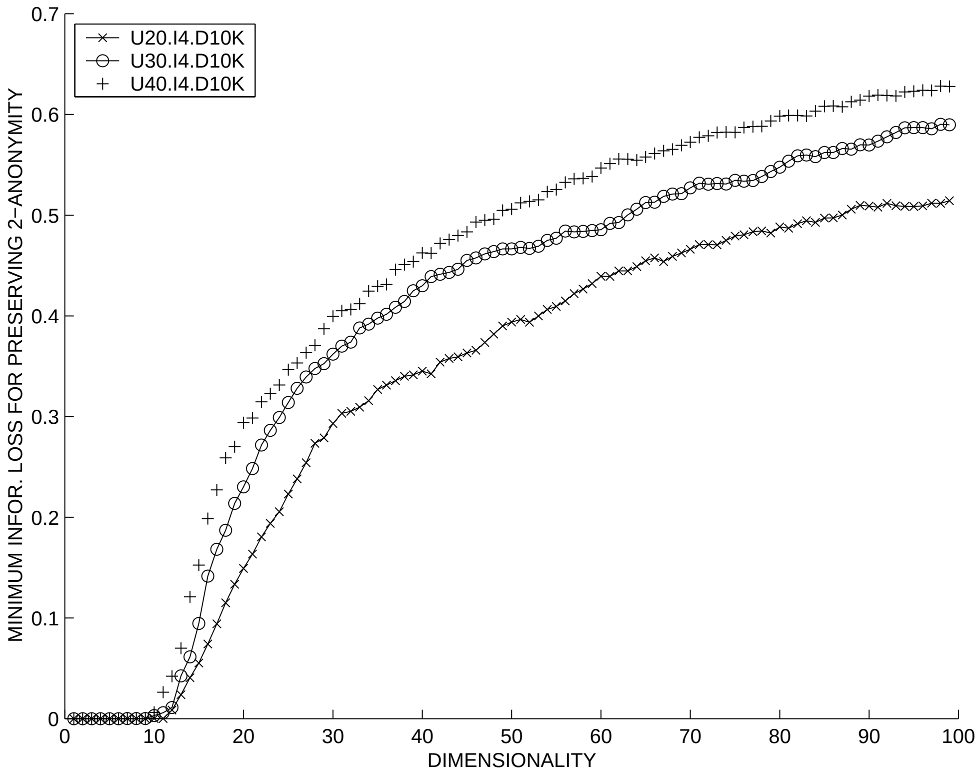

Conclusions and Summary

- Analysis of k-anonymity in high dimensionality.

- Earlier work has shown that k-anonymity is computationally difficult (NP-hard).

- This work shows that in high dimensionality, even the usefulness of k-anonymity methods becomes questionable.